

6.2 – Real World Functions

Example 1: The human cannonball

The Great Gonzo, the human cannonball, defies death daily by being fired straight up out of a circus cannon at a muzzle velocity of 30 metres per second. Great Gonzo's flight is actually the result of two forces – the upward propulsion caused by being fired from the cannon, and the downward force of gravity.

The distance, d , traveled by an object moving at a constant velocity, V_o , is $d = V_o t$, where t indicates time. The distance, d , travelled by an accelerating object is $d = 0.5gt^2$, where g is the acceleration due to gravity and t is time. Assume that all objects on earth, not subject to air resistance, will fall at an acceleration of -9.8 m/s^2 .

Gonzo's flight can be studied with the help of a mathematical model. You will need three functions: one for the cannon, one for gravity, and one for the combined interaction of both.

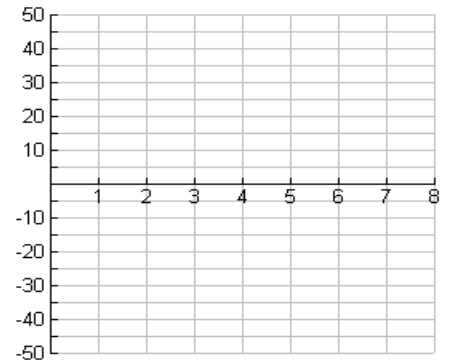
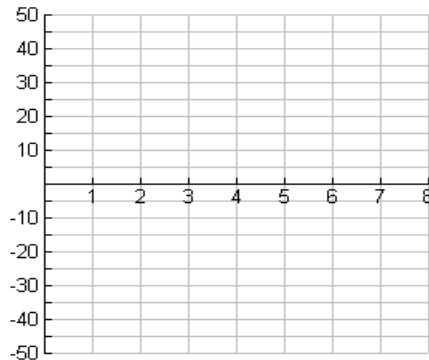
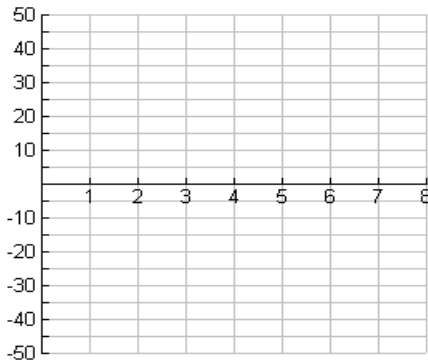
First, set up a window for the graph. X interval from (0 to 8)
Y interval from (-50 to 50)

Define Y_1 to be the distance travelled upward as a result of the muzzle velocity. Enter $Y_1 = 30X$
Define Y_2 as the distance travelled downward as a result of the acceleration due to gravity. Enter $Y_2 = -4.9X^2$. Sketch the graphs below.

a) $Y_1 = 30X$

b) $Y_2 = -4.9X^2$

c) $Y_3 = Y_1 + Y_2$



In Each case, what does the line $y = 0$ represent?

Now create a new function, Y_3 , by adding Y_1 and Y_2 . Position the cursor in Y_3 . Press **VAR** **)**

ENTER Select 1: Function and enter the function you want to choose, (i.e. Y_1 or ?). $Y_3 = Y_1 + Y_2$.

Turn off Y_1 and Y_2 without erasing them by highlighting the = sign and pressing **ENTER**. Press **GRAPH** and then sketch the graph above.

Press **MODE** and highlight Simultaneous rather than Sequential. This will let you see all three equations graphed simultaneously. Press **Y=** and turn on all three equations by highlighting each equal sign and pressing **GRAPH**. Describe how the graph of Y_3 is affected by the graphs of Y_1 and Y_2

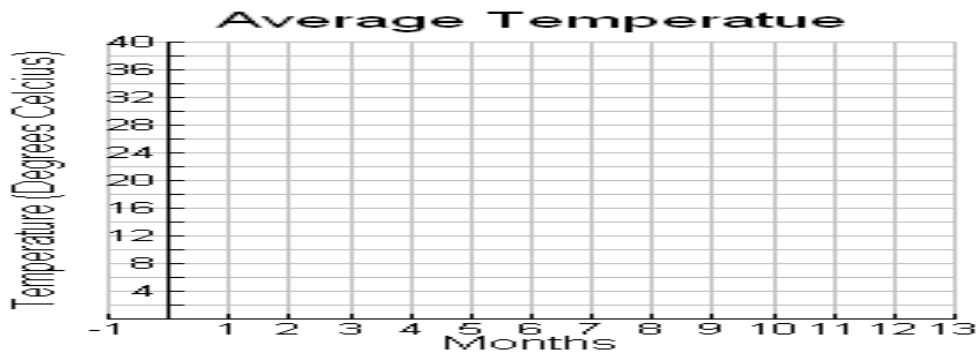
Example 2: Seasonal Temperature Changes

The average temperature in any location changes with the season. In this investigation, you will develop a model for the average temperatures for Victoria, B.C.

Consider the average temperature for the months January (1) through December (12).

Month	1	2	3	4	5	6	7	8	9	10	11	12
Temperature	13.2	15.6	20.5	25.1	28.6	32.4	33.4	33.1	30.5	25.5	20	15.2

- Using the graphing calculator create a STATPLOT using Month in L_1 and Temperature in L_2 and sketch the graph below:



Consider the function $f(x) = a \sin(b(x - c)) + d$

- Write the value of the maximum and minimum temperatures.
- Estimate the vertical translation as $d = \frac{\text{temp max} + \text{temp min}}{2}$
- How does changing d affect the graph of $f(x)$?
- Define the amplitude as $a = \frac{\text{temp max} - \text{temp min}}{2}$
- How does changing a affect the graph of $f(x)$?
- What is the period of the graph?
- Re-define $b = \frac{2\pi}{\text{your period}}$
- How does changing b affect the graph of $f(x)$?
- The horizontal shift for $f(x)$ is c . Adjust c until you have a good fit.
- Make any further adjustments to a, b, c and d to have a good fit. Record the sine function that fits this data. $f(x) =$
- Use sine regression to find the function that best fits this data. Write the function below:

$$f(x) =$$

- How did your function compare to the regression?

6.2 – Real World Functions Practice Questions

Graph each of the following functions and describe;

- domain and range
- Increasing and decreasing intervals
- maximum and minimum values
- does the model seem reasonable

- A catheterization is a special test used to find out how well heart muscles and valves are working. In a normal heart, the concentration of the dye is given by the function $g(x) = -0.006x^4 + 0.14x^3 - 0.053x^2 + 1.79x$, where x is the time in seconds.
- A rectangular area of 60m^2 has a wall as one of its sides. The sides perpendicular to the wall are made of fencing that costs \$6/m, while the side parallel to the wall was made of a more expensive fencing that costs \$8/m. The cost for the fencing is given by the function $C(x) = \frac{12x^2 + 480}{x}$, where x is the length of the rectangle on exposed side.
- A sphere with radius 1 is contained within a cone. The volume of the cone as a function of the base radius, r , of the cone is given by $V(r) = \frac{2\pi r^4}{3(r^2 - 1)}$
- Doctors often use Young's Rule to estimate the proper dosage of a medication for a child when the adult dosage is given. This can be modeled by $C = \frac{y}{y+12} \cdot \text{AdultDose}$, where y is the age of the child and AdultDose represent the amount one would normal give to adult.
- A train traveling at 60mph for a given amount of time, t_1 , and then at 40mph for 8 hours can be modeled by the function $V = \frac{60t_1 + 320}{t_1 + 8}$.
- A rectangular piece of paper with perimeter 30cm is folded into a triangular prism with equilateral triangle ends of length x . The volume of the prism is given by $V(x) = \left(\frac{\sqrt{3}}{4}x^2\right)(15 - 3x)$.
- A rectangular piece of tin with perimeter 50cm is rolled into a cylinder with open ends. The volume of the cylinder as a function of x , the circumference of the circular base, is given by $V(x) = \frac{x^2(25 - x)}{4\pi}$.

Answers Use graphing calculator to graph, see table below for values.

#	Domain	Range	Increasing	Decreasing	Max	Min	Reasonable?
1	$x \in \mathbb{R}$	$x < 203$	$x < 17.5$	$x > 17.5$	203	-	Dye only last 23.5 seconds. Seems short
2	$x \neq 0$	$y \neq 0$	$x < -6.3, x > 6.3$	$-6.3 < x < 6.3, x \neq 0$	-152	152	Yes, assumes less expensive cost in domain
3	$x \in \mathbb{R}$	$y > 0$	$x > 0$	$X < 0$	-	0	As sphere pushes deeper volume increases
4	$x \neq -12$	$y \neq 1$	$X < -12, x > -12$	-	-	-	Yes, does model increase as age increases
5	$x \neq -8$	$Y < 60$	$X < -8, x > -8$	-	-	-	Yes
6	$x \in \mathbb{R}$	$y \in \mathbb{R}$	$0 < x < 3.3$	$x < 0, x > 3.3$	24	0	Yes
7	$x \in \mathbb{R}$	$y \in \mathbb{R}$	$0 < x < 16.7$	$x < 0, x > 16.7$	184	0	Yes, volume increases then decrease as should