6.4A – Composite Functions

Recall that ***composite functions*** are made when two or more functions are applied sequentially.

Total pay is regular “x” plus bonus

Ex. Given two functions *f(x)* and *g(x)* $f \circ g$ means *f[g(x)]*
$g \circ f$ means *g[f(x)]*

Example 1: Consider expressing the fuel cost of a car trip as a function of distance driven. The cars fuel consumption as a function of distance is f(d) = 0.080d, where distance is in kilometers. The fuel cost is a function of the number of litres C(f) = 0.98f, where f is in litres.

We can apply these two functions sequentially such that if one drives 100km

Then f(100) = 0.080 (100) and C(8) = 0.98 (8)
= 8 litres used = \$7.84

As one function then cost as a function of distance can be expressed as C(f(d))

So if drive 100 km C (f(100)) = 0.98 [0.08(100)]
= \$7.84

Example 2: Consider two functions f(x) = 2x + 3 and g(x) = $x^2 – 1$. One can consider two Composite functions f(g(x)) and g(f(x))

Ex. $f(g(x)) = 2(x^2 – 1) + 3$ and $g(f(x)) = (2x + 3)^2 - 1$

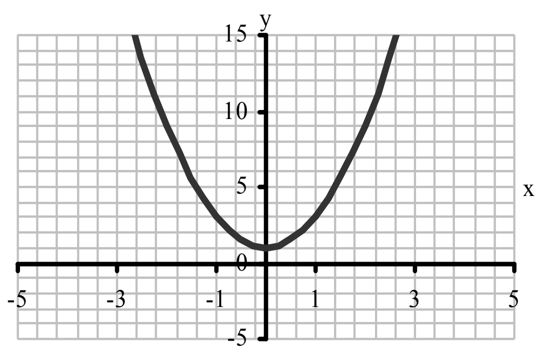
In a table one can see that the order in which the function is composed matters.

x	g(x)	f(g(x))
-2	3	9
-1	0	3
0	-1	1
1	0	3
2	3	9
3	8	19

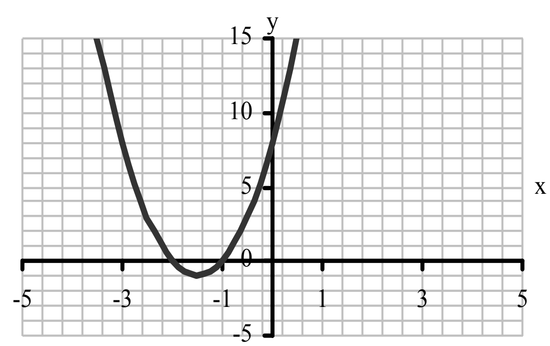
x	f(x)	g(f(x))
-2	-1	0
-1	1	0
0	-	8
1	5	24
2	7	48
3	9	80

Graphically one can see the difference as well.

f(g(x))



g(f(x))



Example 3: Given regular pay is a function of hours worked $p(h) = 8h$ or $p(x) = 8x$
And a \$25 Bonus pay is added to weekly total pay $t(x) = x + 25$

So then given total pay will be a composition of the regular and bonus, we can consider two possible cases.

Order matters.

$$p \circ t = p[t(x)] = 8(x+25) = 8x + 200$$

and

$$t \circ p = t[p(x)] = (8x) + 25 = 8x + 25$$

If work 20 hours in the week then $p \circ t$ given total of \$360
$t \circ p$ gives total of \$185

One is reasonable, the other is not

Example 4: Consider the following table of values

x	0	1	2	3	4	5	6
f(x)	0	2	4	6	8	10	12
g(x)	7	6	**5**	4	3	2	1

a) Explain how to determine the result of f(g(2)).

Find value when g(2)=5 and use this as the input (x-value) for f(5)=?

b) Find $(g \circ f)(1)$, $(f \circ g)(3)$and g(f(0))

gof (1) = g(f(1))
= g(2)
= 5

fog (3) =

g(f(0)) =

c) Is it possible to evaluate g(f(5))? Why or why not?

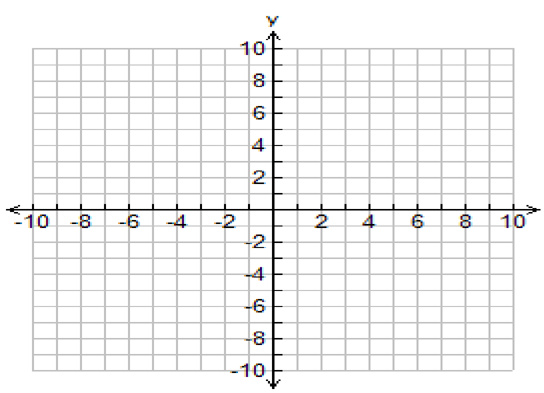
No, the table does not give us a value for g(10).

Example 5: Piecewise function have different parts for different sections. These are not composite nor combination of functions in the strictest sense but as one function is made up or many other functions we might consider them here. This can have applications when different models might apply for different domains (i.e. time)

Ex. Growth might start of exponentially but turn linear after age 5?

Ex. Graph the piecewise function g(x)

$$g(x) = \begin{cases} -2x-5, x < -1 \\ x-3, -1 \le x \le 2 \\ x^2+2, x > 2 \end{cases}$$



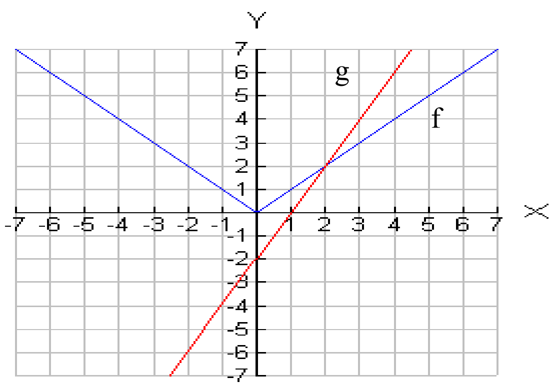
6.4A – Composite Function Practice Questions

1. Use the table to evaluate each expression if possible. If it is not possible explain why.

x	0	1	2	3	4	5	6
f(x)	0	2	4	6	8	10	12
g(x)	7	6	5	4	3	2	1

a) g(f(2)) b) $(f \circ g)(4)$ c) f(g(3)) d) $(g \circ f)(6)$ e) $(g \circ f)(1)$ f) f(g(0))

2. With reference to the graph below;



a) State the domain and range of f an g

b) Use the graph of f and g to evaluate expression, or explain why the expression is undefined or cannot be evaluated.

i) $(f \circ g)(1)$ ii) g(f(1))

iii) f(g(0)) iv) g(f(0))

v) $(g \circ f)(-2)$ vi) $(g \circ f)(5)$

3. If $f(x) = \frac{1}{x}$ and g(x) = 2x – 5, find each of the following, it exists

a) g(-10) b) f(2/3) c) $f\left(g\left(\frac{5}{7}\right)\right)$ d) $(f \circ g)(4)$ e) g(g(0)) f) $f\left(g\left(\frac{5}{2}\right)\right)$

4. Pam buys a pair of shoes and has a coupon for $10 off. The day she busy the shoes, the store as a sale offering 25% off the price of all shoes.

   a) Write a function to represent the cost, C, of a pair of shoes at price, p, if just the coupon reduction is applied to the cost of the shoes. Write a function that represents the cost, D, if just the discount is applied to the cost.
   b) What does the function D(C(p)) represent if a pair of shoes has a regular price of $80? Determine D(C(80))

5. Consider two function f and g. Explain why the domain of f is the range of g for f(g(x))

6. If h(k) = 3k –2, g(t) = 3t + 2 and f(x) = 3x – 2 and C(k) = f(g(h(k))) determine C(4)

7. Refrigeration slows down the growth of bacteria in food. The number of bacteria in a certain food is approximated by $B(T) = 15T^2 - 70T + 600$, where T represent the temperature in degrees Celsius and $3 \le T \le 12$. Once the food is removed from the refrigerator, the temperature T(t) is given by T(t) = 3.5t + 3, where t is the time in hours and $0 \le t \le 3$.

   a) At 1.5 h, about how many bacteria are in the food?
   b) When will the bacteria count reach about 1200?

8. Find expressions for $f \circ g$ and $g \circ f$ for each pair of functions below.

a) $f(x) = \frac{1}{x}$ and g(x) = 4x + 3

b) f(x) = 2x – 3 and g(x) = x + 5

c) $f(x) = \sqrt{x}$ and g(x) = x + 3

d) $f(x) = x^2$ and g(x) = 2x + 5

e) $f(x) = x^3 - x$ and $g(x) = x^2$

f) $f(x) = \sqrt{x^2 + 25}$ and $g(x) = x^4$

9. Given $f(x) = \sqrt{x}$ and g(x) = x+ 4, find

a) f(g(x)) and state the domain and range.
b) g(f(x)) and state the domain and range.

10. A circle has radius r.

a. Write a function for the circle's area in terms of r
b. Write a function for the radius in terms of the circumference, C.
c. Determine A(r(C)).
d. A tree's circumference is 3.6 m. What is the area of the cross section?

11. Express the area of a square as a function of the length of its diagonal.

12. The Volume V of a sphere is a function of its radius r, where $V = \frac{4}{3}\pi r^3$. Express the volume as a function of the diameter.

13. Given f(x) = 2x + 1 find

a) $f \circ f(x)$ b) $f \circ f \circ f(x)$ c) $f \circ f \circ f \circ f(x)$ d) $f \circ f \circ f \circ \ldots \circ f(x)$ (n functions)

14. Determine f(g(x)) and g(f(x)) given f(x) = cos x and g(x) = 2x + 1, state the domain and range of f(g(x)) and g(f(x)), compare f(g(x)) with g(f(x)) algebraically, and verify numerically and graphically with technology.

Answers 1.a) 3 **b)** 4 **c)** 8 **d)** d.n.e. **e)** 5 **f)** d.n.e **2.a)** f: $-7 \le x \le 7$ and g: $-2.5 \le x \le 2.5$ **b) i)** 0 **ii)** 0 **iii)** 2 **iv)** -2 **v)** 2 **vi)** can't be evaluated **3.a)** -25 **b)** 3/2 **c)** -7/25 **d)** 1/3 **e)** -15 **f)** undefined **4.a)** C = p-10, D=.75p **b)** $52.50 **5.a)** 1043 **b)** 1.74 hours **6.** 94 **7.a)** 1043 **b)** 2.51h (-1.75 is inadmissible) use quadratic formula to solve $183.75t^2 - 140t - 810 = 0$ **8.** see table below **9.a)** $\sqrt{x+4}$, D:x>-4, R:y≥0 **b)** $\sqrt{x}+4$, D:x>0, R:y≥4 **10.a)** $A(r)=\pi r^2$ **b)** R(c)=c/2π **c)** $A(R(c))=c^2/4\pi$ **d)** $1.03m^2$ **11.** $A(d)=d^2/2$ **12.** $V=\pi d^3/6$ **13.a)** 4x+5 **b)** 8x+11 **c)** 16x+23 **d)** $2^n x+?$ **14.** see table below

	A	B	C	D	E	F
f(g(x))	$\frac{4x+3}{x}$	2x+7	$\sqrt{x+3}$	$(2x+5)^2$	$X^6 - x^2$	$\sqrt{x^8+25}$
g(f(x))	7	2x+2	$\sqrt{x}+3$	$2x^2+5$	$(x^3 - x)^2$	$(x^2+25)^2$

	Expression	Amplitude	Period	Phase shift	Displacement
f(g(x))	2cos x + 1	2	2π	-	+1

g(f(x))	cos 2(x + ½)	1	π	½	-