6.4B – Composite Functions Problems

1. An airplane is flying at a speed of 350 km/h at an altitude of 1 km and passes directly over a radar station at time t = 0.

    a) Express the horizontal distance, d, in kilometres, that the plane has flown as a function of time, t, in hours.
    b) Express the distance, s, in kilometres, between the plane and the radar station as a function of the distance , d.
    c) Use composition to express s as a function of time and explain what this new function means.
    d) Calculate the distance from the radar station after 90 minutes.

2. A forest fire spears in a roughly circular pattern, with the radius of the burned area increasing at a rate of about 600 m per day. Two week later, an intense rainstorm extinguishes the fire. What area of forest has been burned in this time period?

3. A solar panel is used to power an overhead sign on a highway. Each square metre of solar panel received about 200 W of solar power. This solar panel converts about 15% of the solar energy to electrical power.

    a) Write a function for the solar power S(A), measured in watt (W) for any given area, A, in square metres.
    b) Write a function for the total electrical power, P(S), in watts (W) this solar panel can generate.
    c) How large must the solar panel be if the sign uses 6 W of electrical energy?

4. A furniture salesperson earns 5% commission on his sales, plus a flat rate of $200 per week. Let s represent his average sales per day and a represent his sales in a 5-day week.

    a) Determine an equation for his sales per week.
    b) Determine an equation for his gross wages, w, in dollars, per week, in terms of a.
    c) Substitute the equation from part (a) to determine his gross wages per week in terms of s, his average sales per day.
    d) Determine his gross wages in a week in which he sold an average of $2500 in furniture per day.
    e) If he works on the weekend, he earns 7% commission. Determine his extra wages on a weekend in which he sold $3000 on Saturday and $1500 on Sunday.

5. A supernova explosion creates a spherical shock wave that travels outward at a speed of approximately 3000 km/s

    a) Express the radius, r, in kilometres, of the supernova as a function of time, t, in seconds, if the radius of the supernova before it explodes is 1 000 000 km.
    b) If V is the volume of the supernovas as a function of the radius, find V ∘ r and explain what it represents.
    c) What is the volume of the supernova 15 s after it explodes?

6. A spherical hailstone grows in a cloud. The hailstone maintains a spherical shape while its radius increases at a rate of 0.5 mm/min

   a) Express the radius, r, in millimetres, of the hailstone, as a function of the time, t, in minutes.
   b) Express the volume, V, in cubic millimetres, of the hailstone, in terms of r.
   c) Determine (VBr)(t) and explain what it means.
   d) What is the volume of the hailstone 1h after it begins to form?

7. In the newspaper delivery department, the number of subscribers is x, the number of delivery personnel is approximately by $p = f(x) = \frac{x}{40} + 2$, and the number of supervisors is approximately $s = g(p) = \frac{p}{15}$.

   a) Express the number of supervisors need as a function of x, the number of subscribers
   b) Approximately how many supervisors are needed if there are 4 000 subscribers?

8. A scenic bicycle path is parallel to a canal. The canal and path are 15 m apart. A cyclist travelling at 15 km/h passes a lock (i.e boat gate for moving between different water levels) on the canal at 1:00 p.m.

   a) Express the distance, p, in metres, between the lock and the cyclist as a function f of x, the distance the cyclist has travelled since 1:00 p.m.
   b) Express c as a function g of the time, t, in seconds since 1:00 p.m.
   c) Find f ∘ g. What does this function represent?
   d) How far from the lock will the cyclist be 10 s after 1:00 p.m.?

9. If f(x) = 43x – 3 and $h(x) = 4x^2 - 21$, find a function g such that f ∘ g = h

Answers 1.a) d(t)=350t **b)** s(d)=$(1^2+d^2)^{\wedge}½$ **c)** s(d(t))=$(1^2+(350t)^2)^{\wedge}½$ **d)** 525km **2.** 221.7 km^2 **3.a)** S(A) = 200A **b)** P(S) = 0.15S **c)** A = 0.2m^2 **4.a)** A(s) = 5s **b)** w(a)= 0.05a + 200 **c)** W(s) = 0.25x + 200 **d)** $825 **e)** $315 **5.a)** r = 3000t + 1 000 000 **b)** V = 4B/3(3000t + 1000000)^3 **c)** 4.78 X 10^{18} km^3 **6.a)** r = 0.2t **b)** V = 4/3r^3 **c)** V = 0.032/3Br^3 **d)** 7238 mm^3 **7.** s(x) = (x + 80)/600 **b)** approximately 7 **8.a)** $p = \sqrt{c^2 + 225}$ **b)** c = 4.17t **c)** $p = \sqrt{(4.17t)^2 + 225}$ **d)** 44.32 m **9.** $g(x) = x^2 - \frac{9}{2}$