

### 6.5B – Average and Instant Rate Problems

Scuba divers must not hold their breath as they rise through water or their lungs may burst. This is because the air, which they have breathed to fill their lungs underwater, will expand as the scuba diver rises and the pressure on the body reduces. At every depth, the diver wants 4 litres of air in her lungs for breathing.

If a diver holds her breath, the volume of the air in her lungs varies with the pressure in the following manner:

$$\text{Volume (at new pressure)} = \frac{\text{original volume} \times \text{original pressure}}{\text{new pressure}}$$

The pressure is 1 atmosphere at the surface and increases by 1 atmosphere for every 10 metres below the surface. See table below for first couple of values.

A diver takes a 4-litre breath of air at the surface and descends without breathing. Using the formula above, complete the following table.

Depth (D) in metres	0	10	20	30	40	50	60
Pressure (P) in atmospheres	1	2	3				
Volume (V) of air in lungs in litres	4	2					

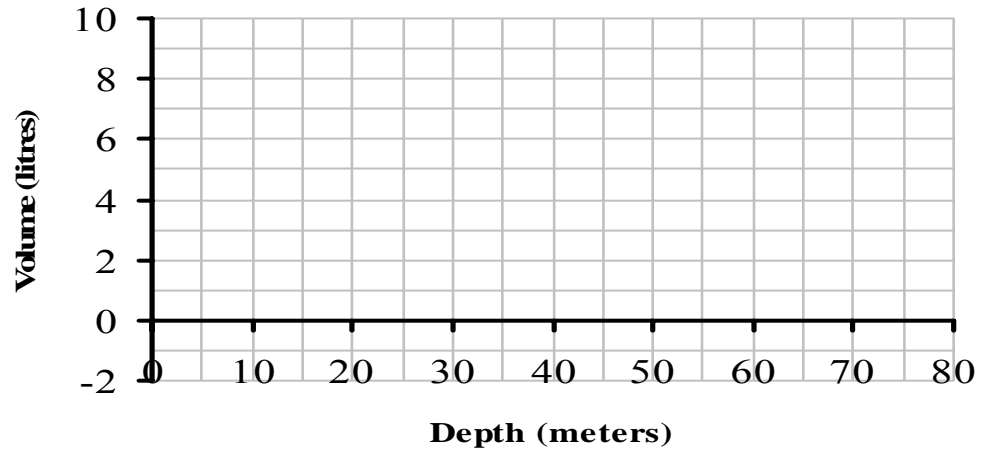
a) Using the answers from the above table, find the relationship between P and D  
Check the rule for D = 20, 30 and 40

b) Using the answers from the above table, find the relationship between V and P  
Check this rule for P = 3, 4 and 5.

c) Use algebra to show that  $V = \frac{40}{d + 10}$

d) Sketch your graph on the grid provided.

**Figure 1: Lung Volume vs Depth for Scuba Diver**



e) Look at your table and your graph and decide through what depths the diver should be most careful about breathing to avoid bursting her lungs.

f) What happens to the graph as the depth increases?

g) What happens to the graph as the depth moves into the negative values?

## 6.5B – Average and Instant Rate Problems Practice Questions

1. The population of bacteria in a sample is 250 000 at 1:00 p.m., 500 000 at 3:00 p.m., and 1 000 000 at 5:00 p.m. Compare methods used to calculate the change in the population and the rate of change in the population between 1:00 p.m. to 5:00 p.m. Is the rate of change constant? Explain your reasoning.
2. Fluorine-20 is a radioactive substance that decays over time. At time 0, the mass of a sample of the substance is 20 g. The mass decreases to 10 g after 11 s, to 5 g after 22 s, and to 2.5 g after 33 s. Compare the average rate of change over the 33-s interval with the average rate of change over consecutive 11-s intervals.
3. In general, does the speedometer of a car measure instantaneous rate of change (i.e., instantaneous speed) or average rate of change (i.e., average speed)? Describe situations in which the instantaneous speed and the average speed would be the same.
4. The distance,  $d$  metres, travelled by a falling object in  $t$  seconds is represented by  $d = 5t^2$ . When  $t = 3$ , the instantaneous speed of the object is 30 m/s. Compare the average speeds over different time intervals starting at  $t = 3$  with the instantaneous speed when  $t = 3$ . Use your observations to select an interval that can be used to provide a good approximation of the instantaneous speed at  $t = 3$ .
5. The height,  $h$  metres, of a ball above the ground can be modelled by the function  $h(t) = -5t^2 + 20t$ , where  $t$  is the time in seconds. Use average speeds to determine the approximate instantaneous speed at  $t = 3$ .
6. The electrical current in a circuit varies with time according to  $c = \frac{3s^3 - s^2 + 5s}{s^3 + 10}$ , where the current,  $c$ , is in amperes, and time  $s$  is in seconds. Find the average rate of change from 0.75 seconds to 1.5 seconds, and find the instantaneous rate of change at 1.5 seconds. Identify any vertical asymptotes.
7. As you get farther from Earth's surface, gravity has less effect on you. For this reason, you actually weigh less at higher altitudes. A person who weighs 55kg can use the function's  $W(h) = \frac{6400(55)}{h + 64000}$  to find their weight,  $W$  in kgs, at a specific height,  $h$  in feet above sea level, above the Earth's surface. Find the average rate of change from heights of 750 ft to 1200 ft above sea level, and find the instantaneous rate of change at 1200 ft above sea level.
8. The pitch  $p$  of a tone and its wavelength  $w$  in meters are related to the velocity  $v$  in m/s of sound through air and can be modeled by the function  $p = \frac{v}{w}$ . For a sound wave with velocity of 985m/s, find the average rate of change in pitch as the wavelength changes from 35m to 45m, and find the instantaneous rate of change at a wavelength of 45m.

9. The average speed of a certain particle in meters per second is given by the equation  $S(t) = \frac{2t^2 + 8t + 5}{t + 3}$ . Find the average rate of change as time changes from 0.5 seconds to 3.5 seconds, and find the instantaneous rate of change at 3.5 seconds.
10. After you eat something that contains sugar, the pH or acid level in your mouth changes. This can be modeled by the function  $L(m) = \frac{-20.4m}{m^2 + 36} + 6.5$ , where  $L$  is the pH level and  $m$  is the number of minutes that have elapsed since eating. Find the average rate of change from 1.5 minutes to 3 minutes, and find the instantaneous rate of change at 3 minutes.
11. In a simple arc for an alternating current circuit, the current at any instant  $t$  is given by the function  $f(t) = 15\sin(60t)$ . Graph the function on the interval  $0 \leq t \leq 5$ . Find the average rate of change as  $t$  goes from 2 to 3. Find the instantaneous rate of change at  $t = 2$ .
12. The weight at the end of a spring is observed to be undergoing simple harmonic motion which can be modeled by the function  $D(t) = 12\sin(60\pi t)$ . Graph the function on the interval  $0 \leq t \leq 1$ . Find the average rate of change as  $t$  goes from 0.05 to 0.40. Find the instantaneous rate of change at  $t = 0.40$ .
13. A Ferris Wheel with a diameter of 50 ft rotates every 30 seconds. The vertical position of a person on the Ferris Wheel, above and below an imaginary horizontal plane through the center of the wheel can be modeled by the equation  $h(t) = 25\sin 12t$ . Graph the function on the interval  $15 \leq t \leq 30$ . Find the average rate of change as  $t$  goes from 24 to 24.5. Find the instantaneous rate of change at  $t = 24$ .
14. The depth of water at the end of a pier in Vacation Village varies with the tides throughout the day and can be modeled by the equation  $D = 1.5\cos[0.575(t-3.5)] + 3.8$ . Graph the function on the interval  $0 \leq t \leq 10$ . Find the average rate of change as  $t$  goes from 4.0 to 6.5. Find the instantaneous rate of change at  $t = 6.5$ .
15. Airbags are a standard safety feature of most cars. Without airbags, a person or object would move forward at the velocity that the car is moving. A person inside a car traveling at 8m/s and weighing 8000kg would follow the function  $v(m) = \frac{8000 - m}{8000 + m} \times 5$ , where  $m$  represents the mass of the person. Find the average rate of change as the mass of the person changes from 57 kg to 75 kg, and the instantaneous rate of change at 75 kg.

**Answers** 1. rate from 1 to 5 = 187500 per hour, double ever 2 hours so exponential model best  $P(t) = 250000(2)^t$   
 2. average  $[0,33] = -0.53\text{g/s}$ , 1<sup>st</sup> 11s  $[0,11] = -0.91\text{g/s}$ , 2<sup>nd</sup> 11s  $[11,22] = -0.45\text{g/s}$ , 3<sup>rd</sup> 11s  $[22,33] = -0.23\text{g/s}$   
 average rate is decreasing because this is exponential decay. 3. Speedometer measures instantaneous speed. Average would equal instant when travelling at a constant speed 4.  $d(2.9) = 42.05$ ,  $d(3.0) = 45$ ,  $d(3.1) = 48.05$  so average from  $[2.9,3.0]$  or  $[3.0,3.1] = 30.5\text{m/s}$ , choosing a closer interval (i.e. 2.99 to 3.00) would give even closer to instantaneous at 3.0 5. at  $t=3\text{s}$ , speed =  $-10\text{m/s}$  6. 0.9629, 1.1 7. -0.0065, -0.0069 8. -0.6254, -0.4864 9. 2.3932, 2 10. -0.373333, -0.2720 11. -12.99, -8 12. 27.5629, 10 13. 1.88, 1.620 14. -0.66756, -0.9 15. -0.0012296, -0.0012