JOHN CABOT CATHOLIC S.S.

Period: ?

Date of Exam: January ?rd, 2008

Start Time: 8:30 am

End Time: 10:30 am

Teacher: Mr. A. Cecchini

Course: MHF 4U1 – ?

Number of Pages: ? (including cover)

Name: _____

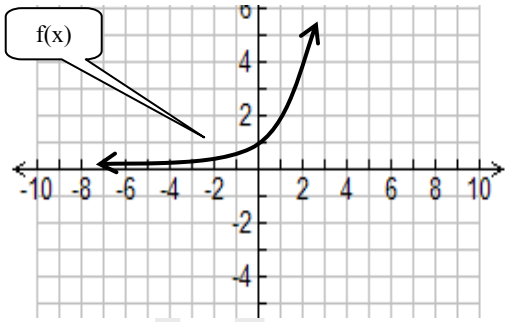
Exam Mark Breakdown		Marks per Category			
		Communication 20%	Knowledge 30%	Application 30%	Thinking 20%
Part A	Communication	/?			
Part B	Knowledge		/?		
Part C	Application			/?	
Part D	Thinking				/?
Totals by Category		**/?**	**/?**	**/?**	**/?**

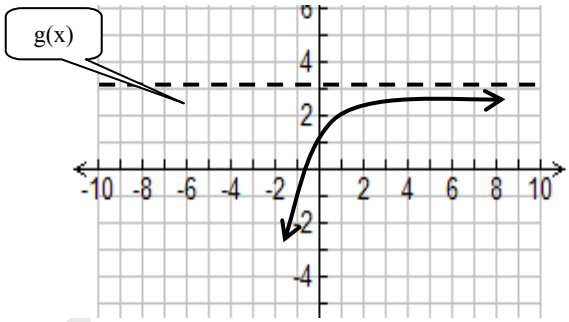
Instructions:
1. Put your **full name** on the line above.
2. Count the number of pages to ensure all are present.
3. Show full solutions where asked.
4. Non-graphing calculators may be used – no sharing will be allowed.
5. Check over answers when finished.

Part A: Communication (? marks)

Instructions: Explain and justify all answers in space provided (marks as indicated)

1. Using the function ***f(x) = x²***, graphically explain the difference between instantaneous and average velocity.
2. Simplify the function $f(x) = \dfrac{6 - 5(3)^{x+1}}{3}$ and then state the domain and range
3. Simplify the function $y = 4(2)^x$
4. Write two equivalent expression to $y = \sin\left(x - \dfrac{\pi}{4}\right)$
5. Using graphs shown, determine the value of f(g(1)).





6. Does $y = x^4$ ever have a negative slope. Explain.
7. Determine the inverse of $y = log_5\, x$
8. Complete the following chart

#	Equation	Horizontal Shift	Vertical Shift	Horizontal Stretch	Vertical Stretch
A	$y = \dfrac{3}{4 - 2x} + 3$				
B	$y = 3\log_5 4x - 1$				
C	$y = -2(3)^{2-x} + 5$				
#	Equation	Phase Shift	Displacement	Period	Amplitude
A	$y = 2\sin 4x + 1$				
B	$y = 3\cos\left(\dfrac{1}{2}(\theta - \pi)\right) - 2$				
C	$y = \dfrac{2}{3}\tan\left[3\left(\theta + \dfrac{\pi}{4}\right)\right] + 1.5$				

Part B: Knowledge, Understanding & Skills (? marks)

Instructions: ***Answer questions #? – ? in space provided (1 mark each = ? marks)***
There are no marks given for rough work on these questions.

9. Determine the exact equation of the following, in form f(x) = k(x - a)(x - b)(x - c)(x + d), given the information below.

   a) f(x) = k(x – 1)(x + 2) and goes through point (-3, 6) _____

   b) g(x) = k(x + 2)(x – 2)(x + 3) and g(1) = -24 _____

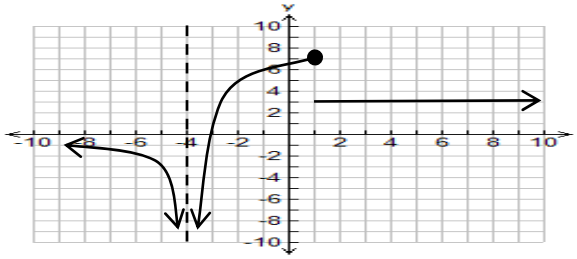
10. Determine the remainder when $(x^3 - 2x^2 + 3x + 4) \div (x + 2)$ _____

11. Determine the exact value of $3^{-1} - 2\left(\frac{1}{8}\right)^{-\frac{2}{3}}$ _____

12. Is $x - 3$ a factor of $x^5 - 4x^3 + x^2 - 3$ _____

13. Evaluate $|3| - |-5| + |3 - 9|$ _____

14. List point(s) of discontinuity in the graph of $y = g(x)$ below. _____



15. Write, $4 = \log_2 16$, in exponential form _____

16. Evaluate ***h(3)*** given, $h(x) = -3x^{-2}\log_3(\sqrt{x^2 + 18})$ _____

17. Evaluate; $\lim_{x \to \infty} \frac{x(2x^2 - 5x)}{2x - x^3}$, if it exists _____

18. Given $f(x) = -3x - 2$, $g(x) = (x+1)^2 + 2$, $m(x) = \frac{1-x}{2}$ determine;

   a) $f(x) + g(x)$ _____

   b) $m(f(x))$ _____

   c) $f(x) \cdot g(x)$ _____

   d) $f(x)/m(x)$ _____

   e) $g[f(2)]$ _____

Instructions: Answer questions #? – ? in the space provided (marks as indicated)

19. Use long division to express the given function $f(x) = (x^2 + 3x - 5)/(x - 1)$ as the sum of a polynomial

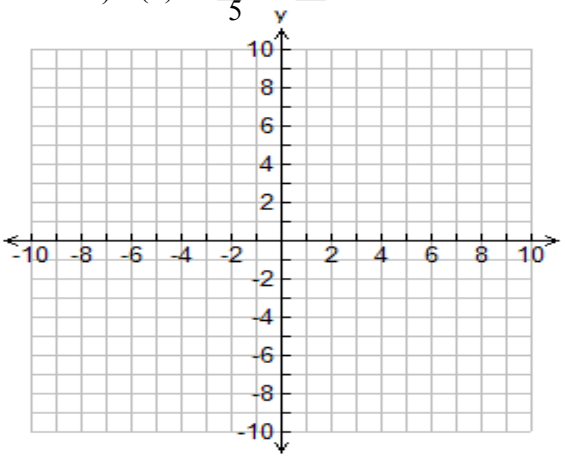
20. Given, $f(x) = \frac{x^3 - 8}{x - 2}$ express *f(a+1)* in simplest form.

21. Complete the following chart

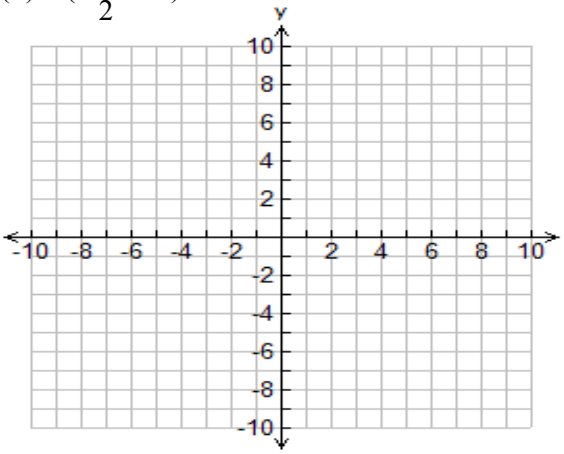
#	Equation	Asymptotes			Intercepts	
		Vertical	Horizontal	Oblique	x-axis	y-axis
A	$y = (x-2)(x+1)(x-3)$					
B	$y = \frac{1}{x+1} + 3$					
C	$f(x) = \frac{x+1}{x^2-4}$					
D	$g(x) = \frac{x^2-9}{x+2}$					

22. Sketch the following functions using most appropriate technique.

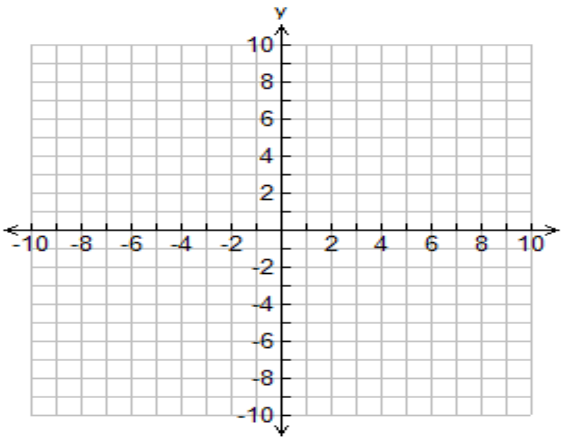
a) $h(x) = -\frac{2}{5}x + 1$



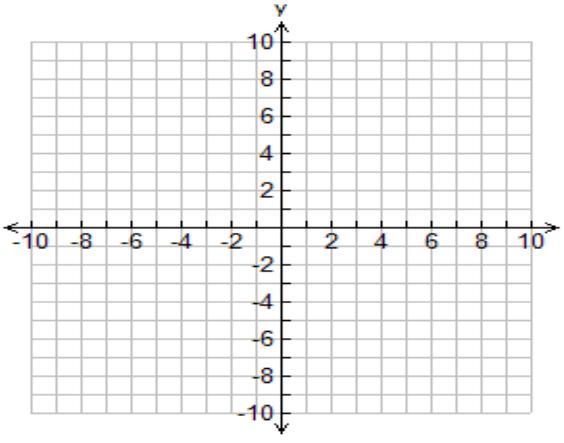
b) $g(x) = (-\frac{1}{2}x - 1)^2 + 2$



c) *m(x) = (2x – 1)(x + 3)*

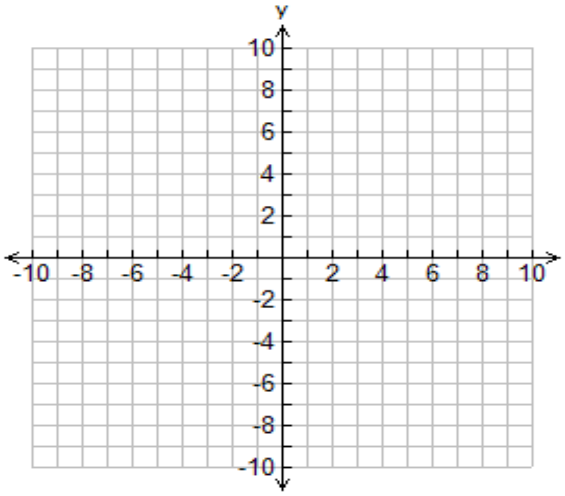


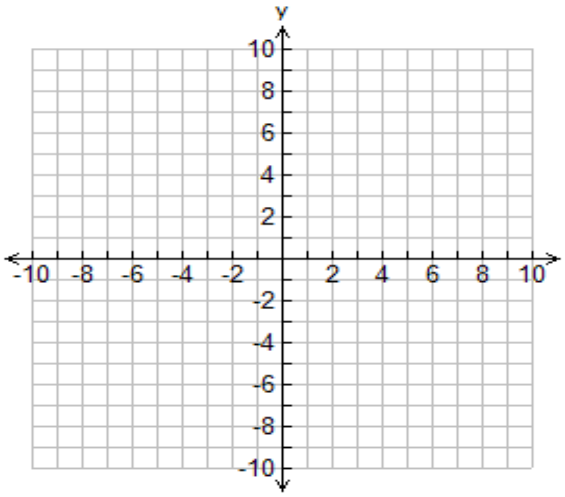
d) $y = (x - 3)^2(x + 5)$



e) $f(x) = -(x-2)^3(x+3)$

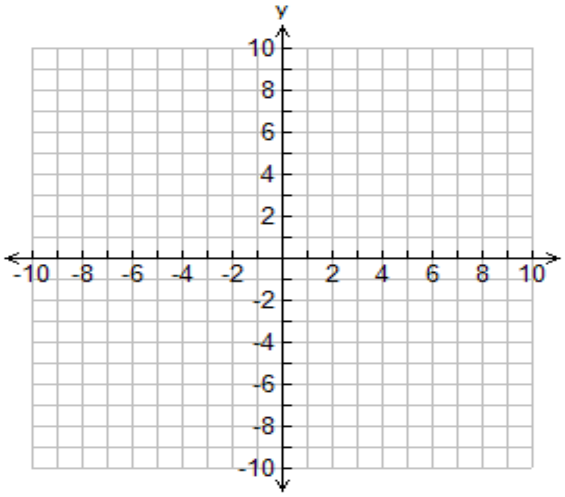
f) $y = \dfrac{1}{(x-2)^2+1}$

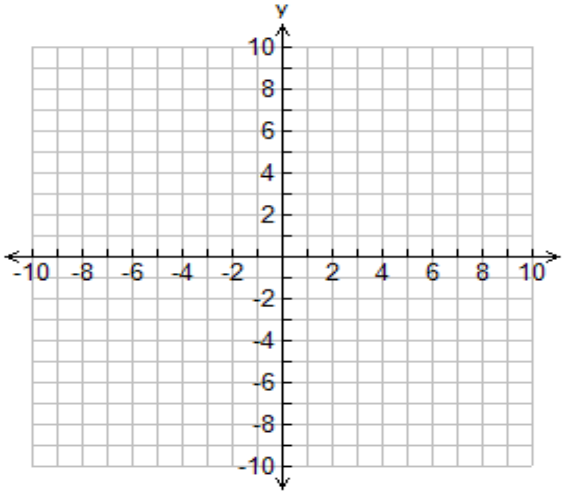




g) $f(x) = \log_3(x+1) - 2$

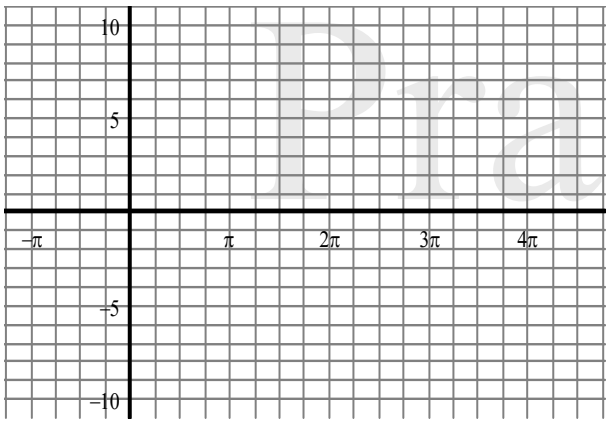
h) $m(x) = 3(\frac{1}{2})^x - 3$

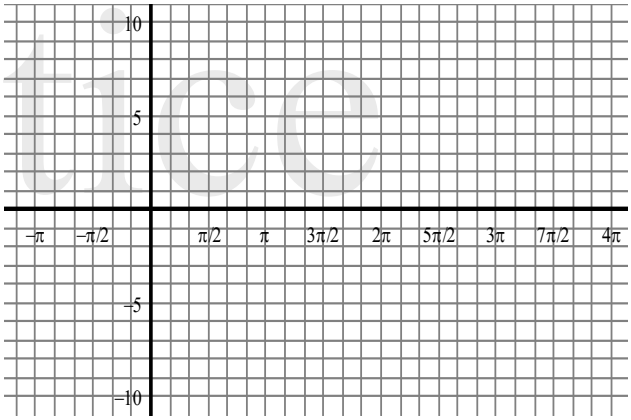




i) $g(x) = \sin 3x - 5$

j) $y = 2\tan(x - \pi) + 2$





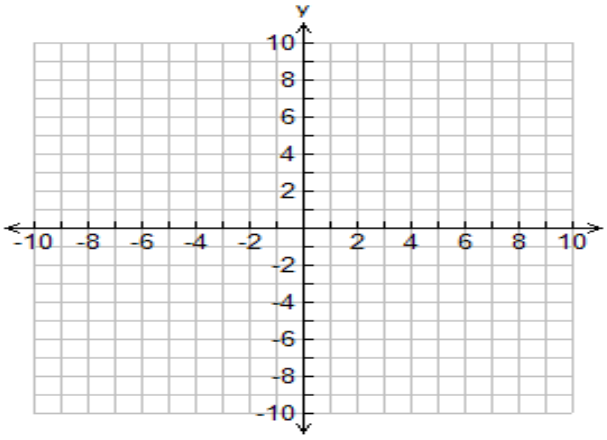
Part C: Application (? marks)

Instructions: Answer all question fully in space provided. Show all step (marks as indicated)

23. Create a function that has a vertical asymptote at x = 3 and x = -2, x-intercept at -1 and horizontal asymptote at y = -5

24. If $\sin x = -\frac{2}{3}$,where $\frac{3\pi}{2} < x < 2\pi$ determine the exact value of cos (2x).

25. Given h(x) = 2x -1 and g(x) = x^2 +1 sketch f(x) = g(x) + h(x)



26. Factor fully a) $x^3 - 27$ b) $6x^3 - 17x^2 + 11x - 2$

27. Solve a) $-x^2 - 5x - 6 < 0$ b) $x^3 - 3x^2 = 4x - 12$

c) $\frac{2+x}{-5} < \frac{2}{3}x + 1$ d) $x^3 - 9x^2 < 24 - 26x$ e) $4x^4 - 2x^3 - 16x^2 = 8x$

28. How long, to nearest tenth of a year, does it take money to double at a rate of 5.25% /a compounded annually?

29. Functions $r(x) = -x^2 + 30x$ amd $c(x) = 17x + 36$ are the estimated revenue and cost functions for the manufacture of a new product. Given profit is revenue minus cost;

a) Determine the average profit function, $AP(x) = \frac{P(x)}{x}$

b) Express the average profit function in a different form.

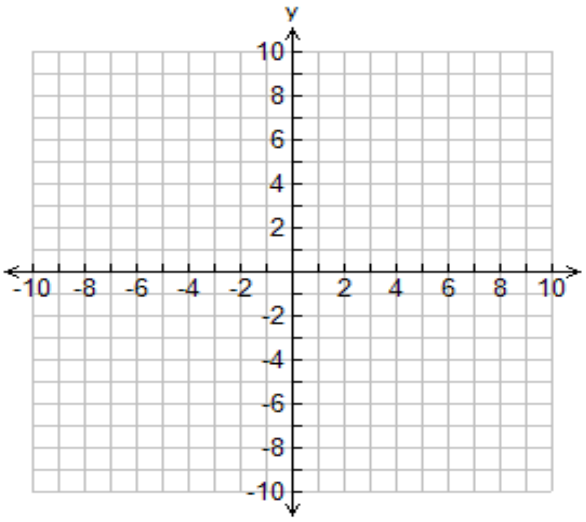
c) What are the break even quantities?

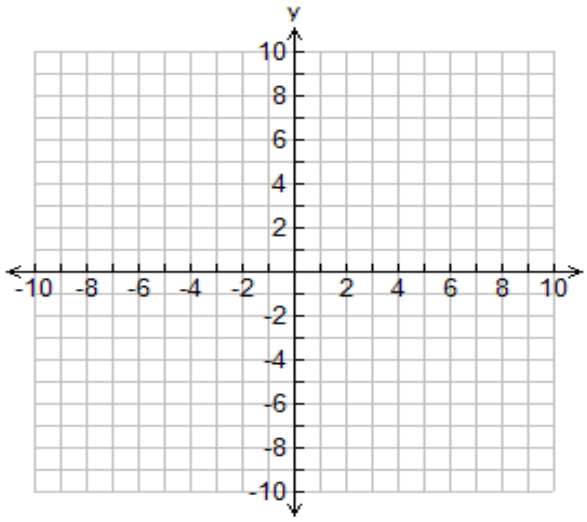
30. When $ax^3 + bx^2 + 4x + 1$ is divided by $x - 1$, the remainder is 12. When it is divided by $x + 2$, the remainder is -20. Find the values of a and b.

31. Analyze and sketch the following

a) $f(x) = \dfrac{2x^2 - 5x - 7}{x - 3}$

b) $g(x) = \dfrac{x^2 - 9}{x^3 + 4x^2 - x - 4}$





Part D: Thinking & Inquiry (? marks)

Instructions: ***Answer all questions in full in the space provided (marks as indicated)***
Provide labeled diagrams where appropriate.

32. Write a cubic function h(x), that has a y-intercept of -6 and an x-intercept of -3, given that $h(1) = 0$ and $h(x) \leq 0$ when $x > -3$.

33. A sperical hailstone grows in a cloud. The hailstone maintains a spherical shape while its radius increases at a rate of 0.5 mm/min.

    a) express the radius, r in millimeters, of the hailstone, as a function of time, t in minutes.

    b) express the volume, V, in cubic millimetres, of the hailstone in terms of r.

    c) Determine v[r(t)]

    d) What is the volume of the hailstone 1h after it begins to form.

34. Bob earns $19.45/h operating a fork lift at Home Depot. He recieves $0.64/h more for working the evening shift, as well as $0.39/h more for working weekends.

    a) Write a function that describes Bob's pay.

    b) What function shows his evening shift premium?

    c) What function show his weekend premium?

    d) What function represents his earnings for the night shift on Saturday?

    e) How much does Bob earn working 11h on Satruday evening, if he earns time and half on that days rate for any hours more than 8 hours work?

35. The table below documents the world population since 1750 in 50 year intervals. Enter the data into your graphing calculator and answer the questions below;

Interval	0	1	2	3	4	5
Population (in billions)	0.79	1.19	1.78	2.68	4.03	6.06

    a) Perform an exponential regression to determine equation that fits data. Give your equation in the form $P(y) = ab^y + c$ where y is the calendar year AD.
    b) Using your equation from part a, predict the world's population, to nearest tenth, in 2071.

Formula Page – MHF 4U1

$m = slope = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$ $a^{-n} = \left(\frac{1}{a}\right)^n$ *or* $\frac{1}{a^n}$ $x^{\frac{a}{b}} = \sqrt[b]{x^a}$ *or* $\left(\sqrt[b]{x}\right)^a$

Growth/decay formula: $A_f = A_o(base)^{\frac{t}{t_i}}$ where A_f = end A_o = original

Logarithms properties Basic: a) $\log_b 1 = 0$ b) $\log_b b = 1$ c) $\log_b b^x = x$ d) $b^{\log_b x} = x$

Operational: a) $\log_a x^r = r \log_a (x)$ b) $\log_a xw = \log_a x + \log_a w$ c) $\log_a\left(\frac{x}{w}\right) = \log_a x - \log_a w$

Formulii that make use of logarithms: $M = \log\left(\frac{I}{I_o}\right)$ $L = 10\log\left(\frac{I}{I_o}\right)$ $pH = -\log\left[H^+\right]$

Arc angle formula $\theta = \frac{a}{r}$ where θ = angle in radians a = arc length

Radian/Degrees conversion formulas 360° = 2π rad or 180° = π rad

Exact trigonometric equations for common angles

x°	0°	30°	45°	60°	90°
x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
Sin x	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos x	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan x	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	-

Co-function relationship $\sin(\frac{\pi}{2} - x) = \cos x$ and $\cos(\frac{\pi}{2} - x) = \sin x$

Supplemental relationship $\sin\theta^\circ = \sin(\pi - \theta^\circ)$ and $\cos\theta^\circ = -\cos(\pi - \theta^\circ)$

Positional relationship $\sin(-x) = -\sin x$ and $\cos(-x) = \cos x$

Compound formulas $\cos(x + y) = \cos x \cos y - \sin x \sin y$ $\sin(x + y) = \sin x \cos y + \cos x \sin y$
$\cos(x - y) = \cos x \cos y + \sin x \sin y$ $\sin(x - y) = \sin x \cos y - \cos x \sin y$

Double angle formulas $\sin(2x) = 2\sin x \cos x$ $\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1$ $\tan(2x) = \frac{2\tan x}{1 - \tan^2 x}$

Quotient Identities $\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$

Reciprocal identities $\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

Pythagorean identities $\cos^2 x + \sin^2 x = 1$ *or* $\cos^2 x = 1 - \sin^2 x$ *or* $\sin^2 x = 1 - \cos^2 x$

Answers

1. Secant gives the average slope between two points. Tangent gives instantaneous slope at one point.
2. $f(x) = 2 – 5(3)^x$ or $f(x) = -5(3)^x + 2$ Domain: $x \in R$ Range: $y < 2$
3. $y = 2^{x+2}$
4. As function is periodic answers may from; $y = \sin(x + 7\pi/4)$ or $y = \cos(x + \pi/4)$
5. $f(g(1)) = 4$
6. Yes. When $x<0$ graph decrease and thus has a negative slope. Illustrate with sketch.
7. $y=5^x$
8. See table below

#	Equation	H Shift	V Shift	H Strecth	V Strecth
A		+2	+3	-½	-3
B		n/a	-1	¼	+3
C		+2	+5	-1	-2
#	Equation	Phase Shift	Displacement	Period	Amplitude
A		n/a	+1	π/2	2
B		+π	-2	4π	+3
C		-π/4	1.5	2π/3	2/3

9. a) $f(x) = 3/2(x-1)(x+2)$ b) $g(x) = 2(x+2)(x-2)(x+3)$
10. $r(x) = -18$
11. -7 2/3 or -7.67
12. no
13. 4
14. $x = -4, 1$
15. $2^4 = 16$
16. -1/2
17. -2
18. a) x^2-x-2 b) $(3x+3)/2$ c) $-3x^3 – 8x^2 – 16x – 8$ d) $(-6x-4)/(x-1)$ e) 51
19. $f(x) = (x+4) - \dfrac{1}{x-1}$
20. $f(x) = a^2 + 4a + 7$
21. See table below

#	Equation	Vertical	Horizontal	Oblique	x-axis	y-axis
A		n/a	n/a	n/a	2, -1, 3	+6
B		-1	y =3	n/a	-4/3	4
C		x = -2, +2	y = 0	n/a	-1	-1/4
D		X = -2	n/a	y= x - 2	3, - 3	-4.5

22. Transformations technique is best used for #a,b,g,h,i, zeros technique for #c,d,e and reciprocal technique for #f. Check sketches using graphing calculator.

23. $f(x) = \dfrac{-5(x+1)^2}{(x-3)(x+2)}$

24. $\cos^2 x - \sin^2 x = 1/9$
25. $f(x) = x(x + 2)$
26. a) $(x - 3)(x^2 + 3x + 9)$ b) $(x - 2)(3x - 1)(2x - 1)$
27. a) $x < -3$ or $x > -2$ b) $x = -2, 2, 3$ c) $x > -3$ d) $x < 2$ or $3 < x < 4$ e) $x = 0$ $x = ½$
28. $t = 13.5$ seconds

29. a) $AP(x) = \dfrac{-x^2 + 13x - 36}{x}$ b) $AP(x) = \dfrac{(x-4)(x-9)}{x}$ *or* $AP(x) = (-x + 14) - \dfrac{50}{x}$ c) 4, 9

30. $a = 4$, $b = 3$
31. Table below gives some characteristics. Check actual sketches on graphing calculator.

#	Equation	Vertical	Horizontal	Oblique	x-axis	y-axis
A		$x = 3$	n/a	$y = 2x + 1$	-1, 1/2	7/3
B		$x = -1, 1, 4$	$y = 3$	n/a	-3, +3	9/4

32. $h(x) = -2(x + 3)(x - 1)^2$

33. a) $r(t) = 0.5t$ b) $v(r) = \dfrac{4}{3}\pi r^3$ c) $v(t) = \dfrac{4}{3}\pi(0.5t)^3$

    d) $V(60) = 113097$ mm^3 or 113 cm^3

34. a) $p(h) = r(h) + e(h) + w(h)$ b) $e(h) = 0.64h$ c) $w(h) = 0.39h$
    d) $p(h) = 20.48h$ e) $p(h) = 20.48(11) + 20.48(11-8) = \$286.72$

35. a) $P(y) = 0.79(1.503)^{\frac{y-1750}{50}}$ b) $P(2071) = 10.81$ billion