_____ % on this test only	/10 Communication	/14 Knowledge	____ Initials
_____ % overall in course	/15 Application	/10 Thinking	

Student (formal name): _____ **January 2011**

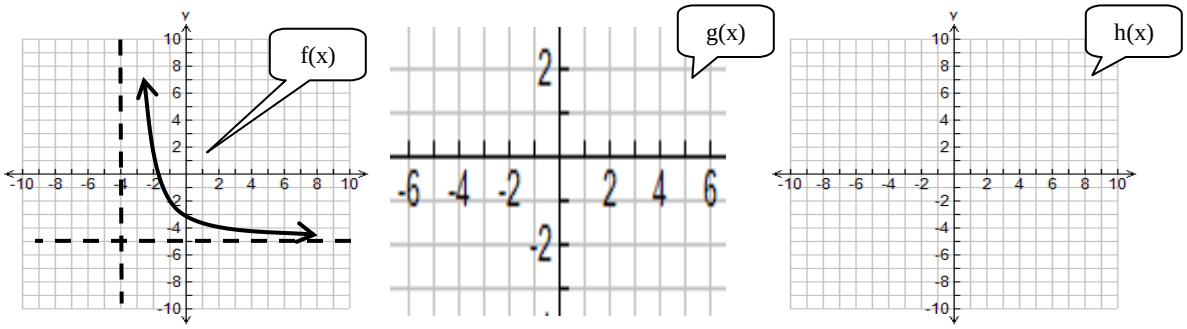
Assignment Set #1 – due January 21th, 2011

General Instructions: *As this is a take home assignment you have a plethora of resources (calculators, computers, textbooks, notes, other people not taking MHF) that you can utilize to help you. Take advantage of these to help you double and triple check your answers. Moreover, give careful thought to your answers and how the final answer should be expressed or written as this is an all or nothing marking scheme. Your work should be well explained and* ***neatly written****. Have someone proof read it for clarity and logical succinctness. Lastly, all work should be your own. Getting help is fine, but plagiarism will not be tolerated.*

Part A: Communication (10 marks)

Instructions: ***Write answers on grid or line provided.***

1. Given $g(x) = 1/f(x)$ and $h(x) = f^{-1}(x)$ sketch both g(x) and h(x) on the grids provided. Clearly indicate any asymptotes and other relevant points.



2. A sinusoidal function has an amplitude of 3 units, a period of 3π/2, and a maximum at (0,4). Represent the function with an equation in two different ways.

   Equation #1: _____ Equations #2: _____

3. The equivalent exponential form of $a = b \log_c d$ is _____

4. Given tan θ= -¾ and θ is an angle is standard with terminal arm is second Quadrant, determine the trigonometric ratio for sin θ. _____

5. Find θ, $\pi \le \theta \le 2\pi$ given $\cos\theta = -\frac{1}{2}$ _____

6. Convert 5π/12 radians to nearest degrees. _____

7. Convert -210°degrees to **exact** radians. _____

8. The exact number of zeros of y = sin 2x, $0 \le x \le 2\pi$ is; _____

Part B: Knowledge, Skills, Understanding **(14 marks)**

Instructions: Place answers to questions #9-22 in the boxes below using CAPITAL letters.

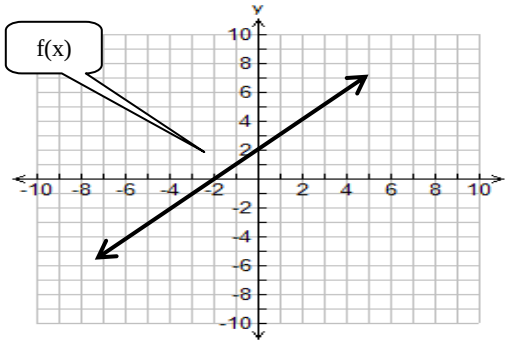
Question #	9	10	11	12	13	14	15	16	17	18	19	20	21	22
Your answer														

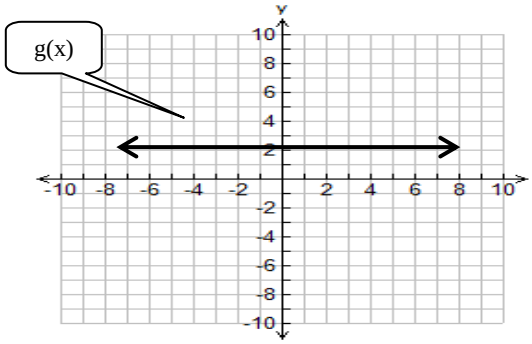
9. Which functions listed below **do not** have a vertical asymptote?

(i) $y = x^2$ (ii) $y = \log x$ (iii) $y = 2^x$ (iv) $y = \tan x$ (v) $y = 1/x$

a) i only b) iii only c) i and iii d) i, iii, and iv e) ii, iv, and v

10. Using graphs shown, determine the value of f(g(1)).





a) 0 b) 1 c) 2 d) 3 e) 4

11. A certain polynomial function is represented by the data below. Determine the type of function;

x	0	1	2	3	4
f(x)	0	2	10	30	68

a) linear b) quadratic c) cubic d) quartic e) none of these

12. Which statement is true for the function ***f(x) = 2 cos x + 1***?

a) $0 \le f(x) \le 3$ b) $-1 \le f(x) \le 3$ c) $-2 \le f(x) \le 1$ d) $-3 \le f(x) \le 0$ e) $-2 \le f(x) \le 2$

13. The value of $\log_a(\log_a a)$ is;

a) 0 b) 1 c) a d) a^2 e) none of these

14. The interval(s) for which $x^2 - 3x + 2 > 0$ is/are;

a) $1 < x < 2$ b) $x < 1$ or $x > 2$ c) $x > 1$ d) $x < 2$ e) $x > 2$

15. Evaluate $\log_2(\sec 1.2)$ to 3 decimal places.

a) 0.392 b) 1.465 c) 2.003 d) 3.909 e) 4.118

16. A bacteria culture is growing according to the function $N(t) = 100(2)^t$, where N(t) is the population after t hours. How long will it take the population to grow to 51200?

a) 8 h b) 9 h c) 10 h d) 11 h e) 12 h

17. Given that $f(x) = \frac{2x}{3}$ evaluate $f^{-1}(6)$

a) 2 b) 4 c) 6 d) 9 e) 12

18. A helium balloon is rising according to the function ***H(t) = 20log(t + 1)***, where h(t) is the height in meters after t seconds. Determine the average rate of increase of height from t = 9 to t = 99 seconds.

a) 1/9 m/s b) 2/9 m/s c) 1/3 m/s d) 4/9 m/s e) 5/9 m/s

19. If $\sin\theta = \cos\theta = \frac{1}{\sqrt{2}}$, determine the value of $\sin(2\theta)$

a) 0 b) 1 c) 2 d) $\sqrt{2}$ e) $\frac{\sqrt{2}}{2}$

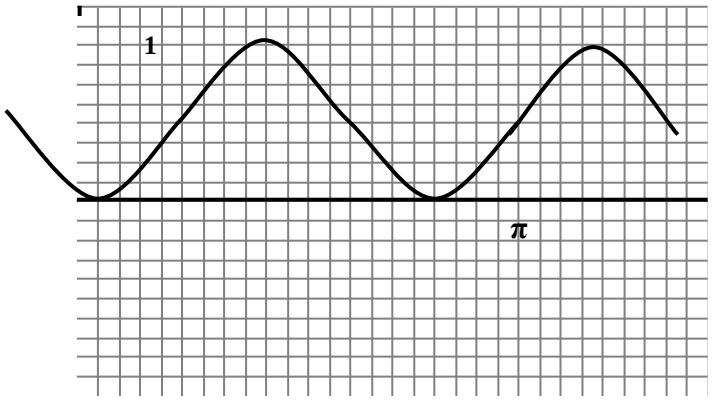
20. Which of the following statements is true given that; $2^a = 6\left(\frac{1}{3}\right)^b$

a) a – b = 1 b) a + b = 1 c) b – a = 1 d) ab = 1 e) a/b = 1

21. The horizontal asymptote for the function $g(x) = \frac{2x^2 - 1}{x^2 + 3}$ is;

a) y = 0 b) y = 1 c) y = 2 d) y = -1/3 e) y = -3

22. Determine the equation for the following. (although sketch might be slightly off one answer works best)



a) y = 0.5sin(0.5x) + 0.5

b) y = 0.5sin(x) + 0.5

c) y = 0.5sin(2x) + 0.5

d) y = 0.5cos(2x) + 0.5

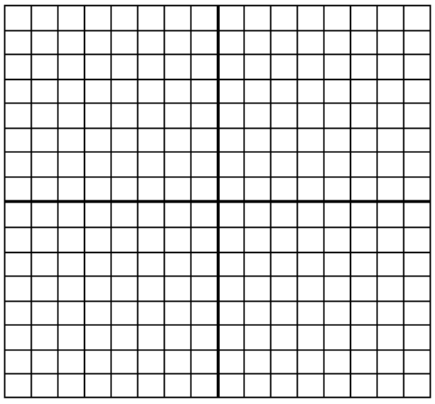
e) y = -0.5sin(2x) + 0.5

Part C: Application **(15 marks)**

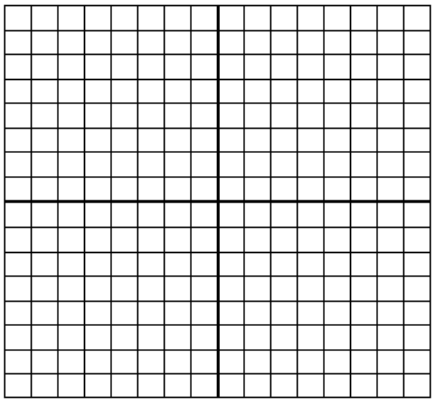
Instructions: *Write your final answers to questions #23 – 32 on the grids or boxes provided. Show any rough work in accompanying space*

23. Sketch the following. Clearly scale axis, label asymptote(s) and any relevant points;

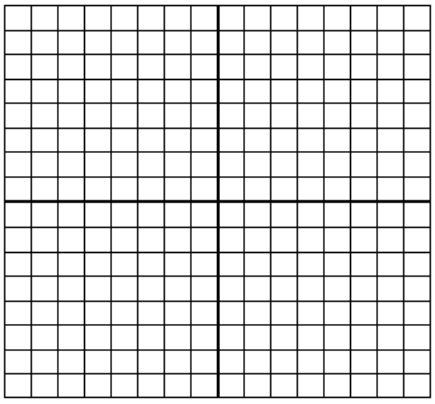
a) $y = \dfrac{1}{2(x-3)^2 - 2}$



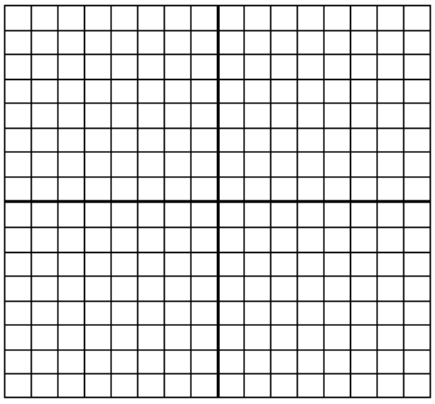
b) $y = \dfrac{x^3 - 6x^2 + 8x}{x^2 - 2x - 3}$



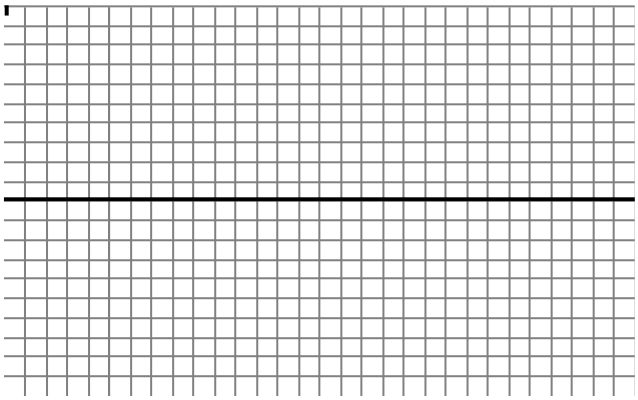
c) $h(x) = -3\left(\frac{1}{4}\right)^x - 2$



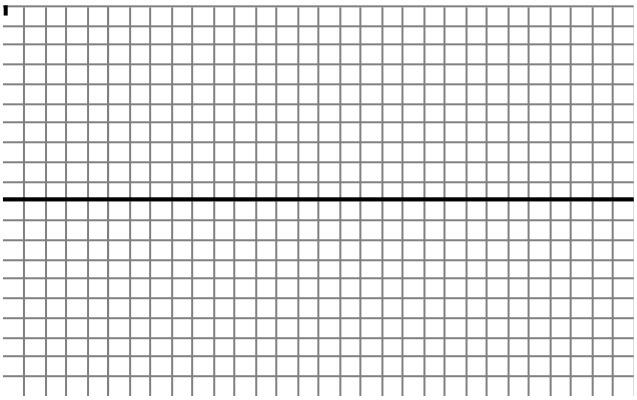
d) $y = -\log_2\left(\frac{1}{2}(x+1)\right) - 3$



e) $y = 2\sin(-2x + \pi/2) - 1$



f) $m(x) = -2\tan\left[0.5\left(x - \frac{\pi}{3}\right)\right] - 4$



24. The height, *h(t)* in meters, of a passenger on a Ferris wheel is given by the function $h(t) = 11 - 10\cos(0.1\pi t)$, where *t* is the time in seconds. How long will it take a passenger to reach a height of 5 meters for the first time? Express answer to 3 decimal places.

25. Divide $x^3 + x + 1$ by $x + 1$ and express answer in for $f(x) = d(x)q(x) + r(x)$

26. An old piece of wood contains 4.2×10^{10} atoms of C^{14} per gram. Carbon from present day wood contains 5.0×10^{10} atoms of C^{14} per gram. If the half-life of C^{14} is 5760 years how old, to nearest year, is the piece of wood?

27. Solve $1 - \tan^2 x = 0$, as **exact** solution, on interval $0 \le x \le 2\pi$;

28. Find the particular member of the family of cubic functions whose x-intercepts are 3, ½ , -2 and passes through the point (2, -24). Write final answer in factored form.

29. A communications satellite completes a circular orbit around the Earth every 6 hours. The satellite is 5,000 km above the Earth, and the radius of the Earth is approximately 6,400 km. How far does the satellite travel in 3 minutes to the nearest km.

30. A 100cm long pendulum swings through an arc length of 20cm. Determine the area that the pendulum swings through.

31. At low tide a cargo ships 10m long unloading ramp slopes down from the ship to the dock making an angle of 30° to the horizontal. High tide comes in and the ship moves closer to the dock changing the angle to 60° to the horizontal. Determine the exact change in the horizontal distance from the ship to the dock from low tide to high tide.

32. Determine the value of $\boldsymbol{k}$ for which the function $f(x) = 2x^3 + 5x^2 + \boldsymbol{k}x - 4$ gives the same remainder when divided by x -1 and x – 2.

Part D: Thinking **(10 marks)**

Instructions: *Write your final answers to questions #33 – 42 on the grids or boxes provided. Show all rough work in accompanying space*

33. Determine an exact polynomial function that can be used to model the function ***f(x)=5 sinx*** on the interval from $0^o < x < 360^o$.

34. Determine the vertical asymptotes of the graph $y = \frac{\sec x}{\log x}$ for $x \leq 2\pi$

35. You are given a solution of hydrochloric acid with a pH of 1.7 and asked to increase the pH of the solution by 1.4. Determine how much (i.e. number of times) you must dilute the solution? Does your answer change if you start with a solution of pH of 2.2?

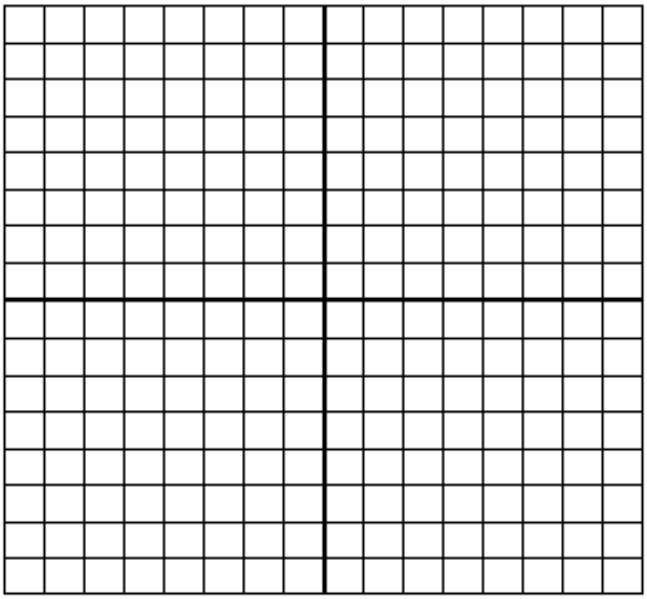
36. Determine the number of points of intersection, for the interval $0 < x < 4\pi$, between the functions $\mathbf{y = 2 \sin \frac{1}{2} x}$ and $\mathbf{y = 3 \cos \frac{1}{3} x}$.

37. The diameter of a helicopter blade is 8m. Given the speed of sound is approximately 1152km/h, find the maximum angular velocity in radians/second and rotations per second so that the blade tips do not exceed the speed of sound.

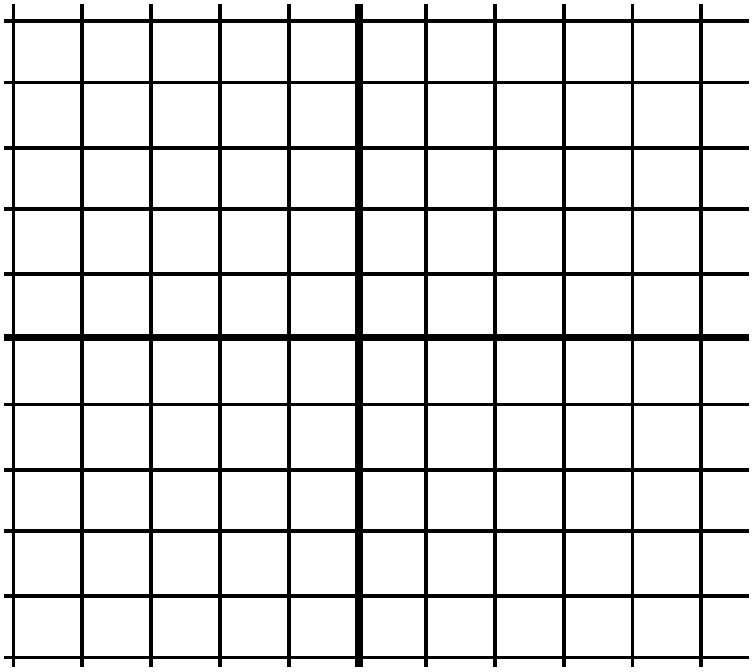
38. The population size, P, of owls (predatory) in a certain region can be modeled by the function $P(t)= 2000 + 500\sin 0.25 (t - \pi/12)$, where t represents the time in months. The population size, p, of mice (prey) in the same region is given by $p(t) = 10000 + 2000\cos (t + \pi/12)$. Determine the time, to nearest tenth of a month, during the first two years when the population of the mice is exactly 5 time the population of the owls. Note that these equation are both in radians measure.

39. The length of daylight cycles periodically approximately every 365 days. If one takes January 1st as day 1, then day 172 (June 21) is the longest day of the year giving about 15.5 h of daylight. Similarly, day 355 (December 21) is the shortest day of the year with about 8.5 h of daylight. Use an algebraic model (i.e. equation you come up with) to calculate on what day(s) of the year we have exactly 12 h of daylight.

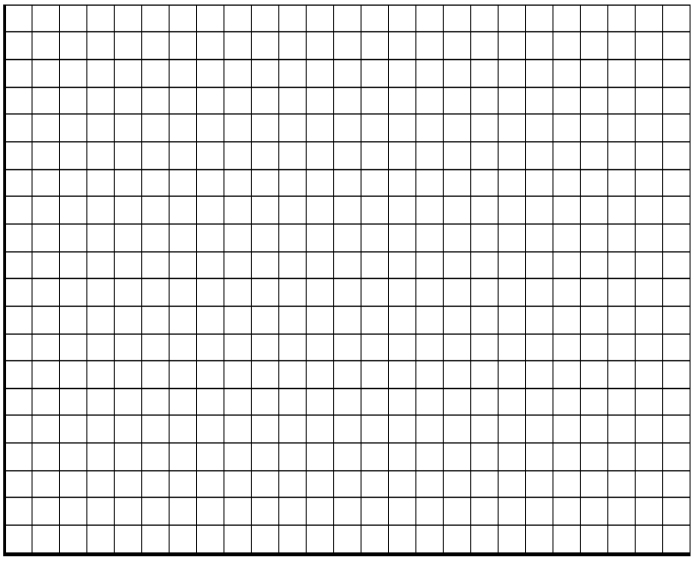
40. Determine algebraically, the exact points (i.e. no decimals) where $y = x^2 - 4$ and its reciprocal would intersect. Provide a rough sketch of sceanrio on grid below.



41. Sketch the reciprocal of $g(x) = x^3 - 4x$ by hand. Show any algebraic work involved and clearly indicate all relevant points.



42. Compare and contrast the two functions $f(t) = 1.2^t$ and $g(t) = 1 + \frac{t^3}{210}$. That is describe how they are similar/different algebraically (what type/classification of function is each one?) and how are they similar/different graphically on the interval $0 < t < 20$. Could each equation be used to model the same situation? Why or why not?



BONUS Question

43. A vibrating string (i.e. guitar) gradually decreasing its displacement over time till it ends up back at the rest position. The strings position from rest can be described with the exponential decay model; $D(t) = A(t)S(t)$, where D(t) is measured in millimetres, A(t) is an exponential function to model the decay portion while S(t) is a sinusoidal function to model the periodic nature of a vibrating string. For $A(t) = a\left(\frac{w}{4}\right)^{\frac{t}{20}}$, ***w*** is the weight (diameter) of the string measured in millimetres and ***t*** is the time measured tenths of a seconds and ***a*** is an adjustable variable for other factors. For $S(t) = \sin k\left(\frac{\pi}{}t\right)$ ***t*** is again measured in tenths of a seconds, and ***k*** is an adjustable variable that affects the period.

a) Assemble the displacement function and manipulate the variables ***a*** and ***k*** so that the maximum amplitude on a string of 0.85 mm diameter is 5.0 mm and the period is such that 15 oscillations take 2.0 seconds. Basically you have to reproduce the graph shown below.

Attach an MS Excel print out or a well draw hand copy of your graph.
Clearly state value of the variables ***a*** and ***k*** that you used as well as the overall function.

b) Use your equation/graph to determine the time it takes for the string to dampen to a vibration of 1.0 mm.

Figure 1: Displacement vs. Time