Lesson 2 – Characteristics of Polynomial Functions

**Terminology**

**Turning Point:** a point on a curve where the function changes from increasing to decreasing, or vice versa.

**Leading Coefficient:** the coefficient of the term with the highest degree in a polynomial

**Absolute Maximum/Minimum:** the greatest/least value attained by a function for all values in its domain.

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**Example 1:**

\[ f(x) = 2x^6 - 12x^4 + 18x^2 + x - 10 \]

- Number of Turning Points: __________
- Leading Coefficient: __________
- Absolute Max/Min: __________

**Odd/ Even Polynomial Functions:**

A function is **even** if it has the property \( f(-x) = f(x) \)

A function is **odd** if it has the property \( f(-x) = -f(x) \)

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**Example 2:** Determine if the following functions are odd, even, or neither.

a) \( f(x) = 2x^7 - 3x^3 + 2x \)  

b) \( f(x) = -2x^6 - 3x^4 \)

c) \( f(x) = -5x^3 + 10x - 1 \)
<table>
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<th>Equation and Graph</th>
<th>Degree</th>
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<tbody>
<tr>
<td>( f(x) = x^4 - 2x^2 + 1 )</td>
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<td>( f(x) = x^3 + 3x^2 - 2x - 5 )</td>
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<td>( f(x) = -\frac{1}{2}x^{10} - \frac{1}{3}x^4 + x^2 )</td>
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<td>( f(x) = -x^3 + x )</td>
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<td>( f(x) = -2x^6 + 3x^4 )</td>
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<td>( f(x) = x^3 - 3x )</td>
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**Summary**

**End Behaviors**

An odd degree polynomial function has opposite end behaviors

If the leading coefficient is positive, then \( as \ x \rightarrow \pm \infty, \ y \rightarrow \pm \infty \)

If the leading coefficient is negative, then \( as \ x \rightarrow \pm \infty, \ y \rightarrow \mp \infty \)

An even degree polynomial has the same end behaviors

If the leading coefficient is positive, then \( as \ x \rightarrow \pm \infty, \ y \rightarrow \infty \)

If the leading coefficient is negative, then \( as \ x \rightarrow \pm \infty, \ y \rightarrow -\infty \)

**Turning Points**

A polynomial function of degree \( n \) has at most \( n - 1 \) turning points

**Number of Zeros**

A polynomial function of degree \( n \) may have up to \( n \) distinct zeros.

A polynomial function of odd degree must have at least one zero.

A polynomial function of even degree may have no zeros.