Momentum & Energy Extra Study Questions

Short Answer

1. What is the momentum of a 1000 kg car moving at 15 m/s [E]?

2. Calculate the momentum of each of the following objects.
   (a) a 0.50 kg ball thrown upward with a velocity of 30 m/s
   (b) a 2000 kg railway car moving south at 10 m/s
   (c) an electron of mass $9.1 \times 10^{-31}$ kg, moving at a velocity of $1.0 \times 10^7$ m/s
   (d) the Earth, of mass $6.0 \times 10^{24}$ kg, moving along its solar orbit with a velocity of $3.0 \times 10^4$ m/s

3. The momentum of a 7.3 kg shot is 22 kg·m/s [forward]. What is its velocity?

4. A bullet travelling at 900 m/s has a momentum of 4.5 kg·m/s. What is its mass?

5. What is the impulse given to a golf ball by a club if they are in contact for 0.0050 s, during which the club exerts an average force of 500 N on the ball?

6. The graph below approximates the force applied to a tennis ball by a racket during the time they are in contact.

   ![Force-time graph](image)

   What impulse does the ball receive from the racket?

7. What impulse is exerted in each of the following cases?
   (a) a force of 25 N [E] on a dynamics cart for 3.2 s
   (b) a hockey stick exerting a force of 120 N on a puck during the 0.05 s they are in contact
   (c) the Earth pulling down on a 12 kg rock during the 3.0 s it takes to fall from a cliff
   (d) a billiard ball bouncing off a cushion, if the force-time graph of the collision appears as below

   ![Force-time graph](image)

   (e) a collision between a toy car and a brick wall, if the force-time graph of the collision appears as below
8. What average force will stop a 1000 kg car in 1.5 s, if the car is moving at 22 m/s?

9. A billiard ball of mass 200 g rolls towards the right-hand cushion of a billiard table at 2.0 m/s and rebounds straight back at 2.0 m/s.
   (a) What is its change in momentum as a result of hitting the cushion?
   (b) What impulse is given to the ball by the cushion?

10. A hockey puck of mass 0.20 kg is sliding along a smooth, flat section of ice at 18 m/s when it encounters some snow. After 2.5 s of sliding through the snow, it returns to smooth ice, continuing at a speed of 10 m/s.
   (a) What is the change in momentum of the puck?
   (b) What impulse does the snow exert on the puck?
   (c) What average frictional force does the snow exert on the puck?

11. A 5000 kg boxcar moving at 5.2 m/s on a level, frictionless track, runs into a stationary 8000 kg tank car. If they hook together in the collision, how fast will they be moving afterward?

12. A 75 kg girl running at 3.0 m/s jumps onto a sled that has a mass of 10 kg and that is already moving in the same direction as the girl, at 2.0 m/s. What will be the final velocity of the girl and the sled, assuming that the sled is on level snow and that there is no friction? (The velocities are relative to Earth.)

13. A 100 g ball moving at a constant velocity of 200 cm/s strikes a 400 g ball that is at rest. After the collision, the first ball rebounds straight back at 120 cm/s. Calculate the final velocity of the second ball. (The velocities are relative to Earth.)

14. A 25 kg object moving with a velocity of 3.0 m/s to the right collides with a 15 kg object moving to the left at 6.0 m/s. Find the velocity of the 25 kg object after the collision, if the 15 kg object does the following.
   (a) continues to move to the left but at only 0.30 m/s
   (b) rebounds to the right at 0.45 m/s
   (c) sticks together with the 25 kg object
   (The velocities are relative to Earth.)

15. The diagram below shows two identical billiard balls before and after a glancing collision.
16. What is the speed of an 1800 kg car with a momentum of $3.0 \times 10^4$ kg·m/s?

17. A $1.5 \times 10^3$ kg car accelerates from rest at $4.0 \text{ m/s}^2$ for 6.0 s.
   (a) What momentum does it acquire in that time?
   (b) What was the impulse exerted on it?

18. A child hits a ball with a force of 350 N.
   (a) If the ball and bat are in contact for 0.12 s, what impulse does the ball receive?
   (b) What is its change in momentum?

19. The average accelerating force exerted on a 5.0 kg shell in a gun barrel is $5.0 \times 10^4$ N, and the muzzle velocity is 200 m/s. Calculate the following.
   (a) the impulse on the shell
   (b) the length of time it takes to move up the heavy gun barrel

20. When a person parachutes, the impact velocity is equal to that attained in free fall from a height of 4.5 m. After contacting the ground, the jumper’s momentum is quickly brought to zero by the Earth. What is
   (a) the impact velocity
   (b) the impulse of the ground on the jumper, assuming a mass of 80 kg
   (c) the average force on the jumper’s feet if (i) he lands stiff-legged and the impulse only last 0.019 s and (ii) he lands with his knees flexed, so that the impulse is extended over a time interval of 0.050 s

21. A toy rocket develops an average forward thrust of 4.0 N when the velocity of the exhaust gases relative to the engine is 30 m/s. Calculate the mass of gases ejected per second.

22. A $1.2 \times 10^4$ kg railroad car is coasting along a level, frictionless track at a constant speed of 25 m/s, when a 3000 kg load is dropped vertically onto the car from above. What will its new speed be, assuming the load stays on the car?

23. A 45 kg boy is running at 4.0 m/s when he jumps onto a 15 kg sled, at rest on a frozen lake. What is the velocity of the boy and sled, if he hangs on?

24. A stationary Volkswagen Beetle of mass $1.0 \times 10^3$ kg is rammed from behind by a Ford Escort of mass $1.2 \times 10^3$ kg, travelling at 20 m/s on an icy road. If they lock bumpers in the collision, how fast will the pair move forward?

25. An arrow travelling at 40 m/s strikes and imbeds itself in a 400 g apple at rest. The apple with the arrow in it moves off horizontally at 10 m/s after the impact. What is the mass of the arrow?
26. A shell of mass 8.0 kg leaves the muzzle of a cannon with a horizontal velocity of 600 m/s. Find the recoil velocity of the cannon, if its mass is 500 kg.

27. On a frictionless air track, a 30 g glider moving to the right at 4.0 cm/s collides with an 80 g glider moving to the left at 1.5 cm/s. If the two gliders stick together in the collision, what is their final velocity?

28. A 125 kg astronaut (with all his equipment) pushes off from his 2500 kg space capsule, acquiring a velocity of 2.0 m/s. (Assume that both astronaut and spacecraft are at rest to begin with.)
   (a) What is the velocity of the space capsule, after he pushes off?
   (b) If he is tethered to the space capsule by a 25 m line, what time will elapse before the line becomes taut?
   (c) Where is the centre of mass of the system when this happens, and what is its momentum?

29. An atom of uranium disintegrates into two particles, one of which has a mass 60 times as great as the other. If the larger particle moves to the left with a velocity of $2.3 \times 10^4$ m/s, with what velocity does the lighter particle move?

30. (a) A polar bear of mass 999.9 kg lies sleeping on a horizontal sheet of ice. A hunter fires a 0.10 kg bullet at the bear with a speed of 1000 m/s. How fast does the bear (with the bullet embedded in a non-vital area) slide after being hit?
   (b) Another polar bear, of mass 990 kg, is wearing a 10 kg bulletproof vest, and is hit by the same hunter. In this case, the bullet bounces straight back with negligible change in speed. How fast does this bear slide after being hit?

31. A 1.5 kg wooden trolley on wheels is stationary on a horizontal, frictionless track. What will be the final velocity of the trolley if a bullet of mass 2.0 g is fired into it with a horizontal velocity of 300 m/s along the direction of the track? (The bullet remains embedded in the trolley.)

32. A truck pushes a car by exerting a horizontal force of 500 N on it. A frictional force of 300 N opposes the car’s motion as it moves 4.0 m. Calculate the work done on the car by the truck.

33. How much work is done on a stone whirled in a horizontal circle at the end of string that exerts a centripetal force of 72 N while the stone moves 0.40 m along its circular path?

34. Calculate the amount of work done in each of the following:
   (a) A gardener pushes down the handle of a lawnmower at an angle of 45° with an applied force of 141 N, while pushing the mower 8.5 m along level ground.
   (b) A 40 kg girl on skates is swung through one revolution in a horizontal circle at the end of a 10 m rope held by a stationary boy at the centre of the circle. The rope has a tension of 50 N in it.

35. The graph shows the magnitude of the horizontal force applied to a 4.0 kg wagon, initially at rest on a horizontal, frictionless surface. Draw a corresponding graph of work done on the object, as a function of its displacement from the starting point.

36. What is the kinetic energy of a rock of mass 12 kg sliding across the ice at 2.0 m/s?
37. What is the speed of an electron in a television tube, if its mass is \(9.1 \times 10^{-31}\) kg, and its kinetic energy is \(9.2 \times 10^{-18}\) J?

38. A sledge hammer has a mass of 4.0 kg and is moving down at a speed of 6.0 m/s when it strikes a fence post, driving it 10 cm farther into the ground.
(a) What was the kinetic energy of the sledge hammer?
(b) What is the average force exerted on the fence post by the hammer?

39. What is the kinetic energy of a wagon of mass 15 kg whose momentum is 30 kg\cdot m/s?

40. What is the momentum of an electron whose kinetic energy is 5.0 keV? (\(m_e = 9.1 \times 10^{-31}\) kg, 1 eV = \(1.6 \times 10^{-19}\) J)

41. Calculate the kinetic energy of each of the following.
(a) an 800 kg automobile moving at 15 m/s
(b) a 2.0 g rifle bullet moving at 500 m/s
(c) a 0.50 kg ball accelerated from rest by a force of 8.0 N for 3.0 m
(d) a stone of mass 0.25 kg being whirled in a circle of radius 2.0 m with a period of \(\frac{\pi}{4}\) s

42. A baseball of mass 250 g, pitched with a speed of 40 m/s, is caught by the catcher, whose glove moves backward 0.25 m while stopping the ball.
(a) What was the kinetic energy of the ball?
(b) How much work did the catcher’s glove do on the ball?
(c) What average stopping force was exerted on the ball?

43. The force-separation graph for a linear collision between a 5.0 kg cart moving initially at 2.0 m/s and a 3.0 cart at rest, is shown below.

![Force-separation graph]

(a) What is the total initial kinetic energy of the two carts?
(b) What is the total kinetic energy of the two carts at a separation distance of 0.05m?
(c) What happened to the remaining initial kinetic energy?

44. A 10 kg block is pushed from rest along a horizontal, frictionless surface with a horizontal force given by the graph below.
(a) How much work is done in moving the block the first 2.0 m?
(b) What is the block’s kinetic energy after it has moved 3.0 m?
(c) What is its velocity at the 3.0 m mark?
45. What is the momentum of a 5.0 kg briefcase with a kinetic energy of \(5.0 \times 10^2\) J?

46. What constant force is required to stop a 1000 kg car travelling at a velocity of 20 m/s in a distance of 1.5 m? Compare this force to the force of gravity on the car.

47. What is the kinetic energy of an electron of mass \(9.1 \times 10^{-31}\) kg in a TV picture tube, if it hits the screen with a velocity of \(1.0 \times 10^7\) m/s?

48. Calculate the amount of work done by the appropriate agent in each of the following cases:
   (a) A workman exerts a horizontal force of 30 N to push a 12 kg table across a level floor a distance of 4.0 m.
   (b) A horse pulls a sled 12 m along the ground at a constant speed of 2.0 m/s against a frictional force of 1500 N.
   (c) A man exerts a force of 150 N, parallel to a slope, to slide a 50 kg mass 8.0 m up the slope.
   (d) A 1.6 kg block is lifted vertically at a constant speed of 1.2 m/s through a height of 20 m.
   (e) A girl pushes a skateboard forward with a thrust of 120 N against the frictional force of the sidewalk of 40 N, while moving 2.5 m.

49. From what height must a 10 kg hammer fall in order to do 240 J of work on a stake being driven into the ground? (Hint: The gravitational force must first do 240 J of work on the hammer.)

50. A 20 kg sled carrying a 40 kg girl is sliding at 12 m/s on smooth, level ice, when it encounters a rough patch of snow.
   (a) What is the initial kinetic energy of the girl and sled?
   (b) If the rough ice exerts an average opposing force of 540 N, in what distance does the sled stop?
   (c) What work is done by the rough ice in stopping the sled?

51. A horizontal force of 50 N is applied to a 2.0 kg trolley, initially at rest, and it moves a distance of 4.0 m along a level, frictionless track. The force then changes to 20 N and acts for an additional distance of 2.0 m.
   (a) What is the final kinetic energy of the trolley?
   (b) What is its final velocity?

52. An applied force of 20 N accelerates a block across a level, frictionless surface from rest to a velocity of 8.0 m/s in a time of 2.5 s. Calculate the work done by this force.

53. A spring whose force constant is 48 N/m has a 0.25 kg mass suspended from it. What is the extension of the spring?

54. What force is necessary to stretch a spring whose force constant is 120 N/m by an amount of 30 cm?

55. A spring with a force constant of 600 N/m is used in a scale for weighing fish. What is the mass of a fish that stretches the spring 7.5 cm from its normal length?
56. A spring in a pogo stick is compressed 12 cm when a 40 kg boy stands on the stick. What is the force constant for the pogo stick’s spring?

57. The force applied to a dynamics cart is measured with a stretched spring. What is the acceleration of a 2.0 kg cart on a flat, frictionless surface if pulled by a spring, of force constant 40 N/m, stretched by a constant amount of 8.0 cm?

58. What is the elastic potential energy stored in a spring whose force constant is 160 N/m when it is compressed 8.0 cm?

59. A block of mass 2.5 kg is sliding across a smooth, level surface at 3.0 m/s when it hits a stationary spring bumper, fixed at one end as shown, whose force constant is 360 N/m. By what amount does the block compress the spring, before coming to rest?

60. What is the force constant of a Hooke’s Law spring if the extension of the spring is 0.15 m when 0.72 J of potential energy is stored in it?

61. How much work must be done on a spring with a force constant \( k = 80 \text{ N/m} \) to stretch the spring 20 cm?

62. How much would a spring scale with \( k = 120 \text{ N/m} \) stretch, if it had 3.75 J of work done on it?

63. A 5.0 g pellet is placed in the barrel of a toy gun and is propelled by a spring of force constant 50 N/m that has been compressed 20 cm and then released. Calculate the maximum velocity of the pellet when shot horizontally.

64. The force-deformation graph for a non-Hooke’s Law spring is shown.

![Force-deformation graph](image)

(a) How much work must be done to compress the spring 0.16 m?
(b) How much potential energy is stored in the spring at this compression?
(c) What speed would a 1.0 kg mass acquire if it were placed next to this compressed spring, on a smooth, horizontal surface, and then released?

65. The distance from the Sun to Earth varies from \( 1.47 \times 10^{11} \text{ m} \), at perihelion (closest approach), to \( 1.52 \times 10^{11} \text{ m} \) at aphelion (farthest distance away). The Sun’s mass is \( 1.99 \times 10^{30} \text{ kg} \); Earth’s mass is \( 5.98 \times 10^{24} \text{ kg} \).

(a) What is the maximum change in Earth’s gravitational potential energy during one orbit of the Sun?
(b) At what point in its orbit is Earth moving its fastest, and what is its maximum change in kinetic energy?

66. The Moon is an Earth satellite of mass \( 7.35 \times 10^{22} \text{ kg} \), whose average distance from Earth’s centre is \( 3.85 \times 10^{8} \text{ m} \).
(a) What is the gravitational potential energy of the Moon with respect to Earth?
(b) What is its kinetic energy and velocity in Earth orbit?
(c) What is its binding energy to Earth?

67. A 2.5 kg roast of beef is suspended from a vertical spring in a butcher’s scale whose force constant is 200 N/m.
(a) What is the extension of the spring?
(b) How much energy is stored in the spring?

68. A linear elastic spring can be compressed 10.0 cm by an applied force of 5.0 N. A 4.5 kg crate of apples, moving at 2.0 m/s, collides with this spring, as shown. What will be the maximum compression of the spring?

69. A 30 kg girl goes down a slide at an amusement park, reaching the bottom with a velocity of 2.5 m/s. The slide is 10.0 m long and the top end is 4.0 m above the bottom end, measured vertically.
(a) What is her gravitational potential energy at the top of the slide, relative to the bottom?
(b) What is her kinetic energy when she reaches the bottom?
(c) How much energy is lost due to friction?
(d) Calculate the average frictional force acting on her as she goes down the slide.

70. A 50 kg girl climbs a staircase consisting of 15 steps, each with a riser 20 cm high. Calculate her change in gravitational potential energy as a result.

71. A spring bumper, whose force-compression relationship is given by \( F = 50x \), is compressed 0.20 m.
(a) What is the potential energy stored in the spring at a compression of 0.20 m?
(b) The spring is compressed further to 0.60 m. What will be the change in elastic potential energy?
(c) A 0.40 kg cart is placed against the compressed spring on a horizontal, frictionless plane, and the system is released. With what velocity will the cart leave the spring?

72. A space vehicle, launched as a lunar probe, arrives at the upper limit of Earth’s atmosphere. At this point, its kinetic energy is \( 5.0 \times 10^9 \) J and its gravitational potential energy is \( -6.4 \times 10^9 \) J. What is its binding energy?

73. A uniform bar of iron is supported by a long, uniform Hooke’s Law spring as shown in A. The spring is cut exactly in half and the two pieces are used to support the same bar, as shown in B. If the whole spring stretched by 4 cm in A, by how much would each half spring stretch in B?
74. A frictionless disc of mass 0.50 kg is moving in a straight line across an air table at a speed of 2.4 m/s when it bumps into an elastic band stretched between two fixed posts. If the elastic band exerts an average opposing force of 1.4 N on the disc for 1.5 s, what will be the final velocity of the disc?

75. A 2.0 kg skateboard is rolling across a smooth, flat floor when a small girl kicks it, causing it to speed up to 4.5 m/s in 0.50 s without changing direction. If the average force exerted by the girl on the skateboard in its direction of motion was 6.0 N, with what initial velocity was it moving?

76. A loaded railway car of mass 6000 kg is rolling to the right at 2.0 m/s when it collides and couples with an empty freight car of mass 3000 kg, rolling to the left on the same track at 3.0 m/s. What is the speed and direction of the pair after the collision?

77. Calculate the recoil velocity of an unconstrained rifle of mass 5.0 kg after it shoots a 50 g bullet at a speed of 300 m/s, with respect to Earth.

78. A 1.0 kg ball moving with a velocity of 2.0 m/s to the right collides straight-on with a stationary 2.0 kg ball. After the collision, the 2.0 kg ball moves off to the right with a velocity of 1.2 m/s. What is the velocity of the 1.0 kg ball after the collision?

79. A competition is held between two teams of physics students, each team made up of three members, and each member having a mass of 60 kg. The teams will take turns climbing onto a cart and jumping off. They want to see whose cart will be moved fastest by propelling it with a jump. They will start at the west end of a cart of mass 120 kg that is free to roll without friction on a level, east–west track. The plan is to run east along the cart and then jump off with a velocity of 10 m/s, with respect to the cart. The first team decides that its three members will run and jump off together. The second team decides that one member will depart according to the rules, followed by the second, and then, finally, the third. Calculate the final velocity of the cart for each team, and discuss the results. (Assume that vectors to the east are positive and omit vector notation.)

80. An experimental rocket sled on a level, frictionless track has a mass of $1.4 \times 10^4$ kg. For propulsion, it expels gases from its rocket engines at a rate of 10 kg/s and at an exhaust speed of $2.5 \times 10^4$ m/s relative to the rocket. For how many seconds must the engines burn in order that the sled acquire a velocity of 50 m/s starting from rest? You may ignore the small decrease in mass of the sled and the small speed of the rocket compared to the exhaust gas. (The velocities are relative to Earth.)

81. A bomb initially at rest on a smooth, horizontal surface is exploded into three pieces. Two pieces fly off horizontally at a 60° angle to each other, a 2.0 kg piece at 20 m/s and a 3.0 kg piece at 12 m/s. The third piece flies off horizontally at 30 m/s.
(a) Determine the direction of motion of the third piece.
(b) What is its mass?

82. The drawings below show glancing collisions between a moving ball and a stationary target ball of equal mass. In each drawing, part of the path of one of the balls has been blacked out. Use the Law of Conservation of Momentum to reconstruct the part that has been deleted, by finding the position of the missing ball at each flash of the strobe.
(a) Find the direction of the path taken by the target ball after the collision, and the distance between its images in the diagram.
83. A golf club exerts an average force of $7.2 \times 10^3$ N on a ball for the $5.0 \times 10^{-4}$ s they are in contact.

(a) Calculate the impulse of the impact on the ball.

(b) If the ball has a mass of 45 g, what velocity will it have as it leaves the club face?
84. A 200 kg shot is discharged horizontally from a cannon, of mass $2.0 \times 10^4$ kg, with a speed of 250 m/s relative to the ground.
   (a) Find the steady force which, acting on the cannon, will stop its recoil in 2.0 s.
   (b) How far will the cannon recoil?

85. A 300 g ball is struck by a bat with an impact that lasts 0.020 s. If the ball moves through the air towards the bat at 50 m/s and leaves at 100 m/s in the opposite direction, calculate the average force exerted by the bat on the ball.

86. (a) A girl on skis (total mass 60 kg including skis) reaches the bottom of a hill moving at 20 m/s. What is her momentum?
   (b) She encounters deeper snow and stops in 3.0 s. What average force does the deeper snow exert on the girl?
   (c) How far into the deeper snow does she penetrate, assuming that the snow exerts a constant force on her?

87. A bullet of mass 0.050 kg, moving with a velocity of 400 m/s, penetrates a distance of 0.10 m into a wooden block firmly attached to Earth. If the resisting force is assumed constant, calculate
   (a) the acceleration of the bullet
   (b) the average force exerted on the bullet
   (c) the time required for it to stop
   (d) the impulse of the collision
   (e) the original momentum of the bullet

88. An 8.0 g bullet, moving at 400 m/s, goes through a stationary block of wood in $4.0 \times 10^{-4}$ s, emerging at a speed of 100 m/s.
   (a) What average force did the wood exert on the bullet?
   (b) How thick is the wood?

89. A 24 g bullet is fired horizontally, embedding itself in a 10 kg block initially at rest on a horizontal ice surface. The block slides along the ice, coming to rest in 2.0 s at a distance of 60 cm from its original position. Assuming that the frictional force stopping the block was constant, calculate the velocity of the bullet.

90. A 2000 kg car travelling east at 24 m/s enters an icy intersection and collides with a 3600 kg truck travelling south at 10 m/s. If they become coupled together in the collision, what is their velocity immediately after impact?

91. A nucleus, initially at rest, decays radioactively. In the process, it emits an electron horizontally to the east, with momentum $9.0 \times 10^{-21}$ kg·m/s and a neutrino horizontally to the south, with momentum $4.8 \times 10^{-21}$ kg·m/s.
   (a) In what direction does the residual nucleus move?
   (b) What is the magnitude of its momentum?
   (c) If the mass of the residual nucleus is $3.6 \times 10^{-25}$ kg, what is its recoil velocity?

92. A steel ball of mass 0.50 kg, moving with a velocity 2.0 m/s, strikes a second ball of mass 0.30 kg, initially at rest. The collision is a glancing one, causing the first ball to be deflected by an angle of 30°, with a speed of 1.50 m/s. Determine the velocity of the second ball after the collision, giving both its speed and direction.

93. A 3000 kg space capsule travelling in outer space with a velocity of 200 m/s. In an effort to alter its course, it fires a 25.0 kg projectile perpendicular to its original direction of motion at a speed of 2000 m/s. What is the new speed of the space capsule and by what angle has its direction changed?
94. A 70 kg boy sits in a 30 kg canoe at rest on the water. He holds two cannonballs, each of mass 10 kg. He picks them up and throws both together over the stern of his canoe. The two balls leave his hands with a velocity of 5.0 m/s relative to the canoe. A 50 kg girl sits in a 50 kg canoe, also at rest on the water. She also holds two 10 kg cannonballs. However, she throws them over the stern of her canoe one at a time, each ball leaving her hands with a velocity of 5.0 m/s relative to the canoe. Assuming negligible friction between the water and the canoe (a poor assumption), calculate the final velocity for each canoe.

95. A hunter shoots a 500 g arrow at a 2.0 kg bird perched on a tall tree growing on flat, level ground. The arrow is launched from ground level with a speed of 40 m/s at an angle of 30° above the horizon. It is travelling horizontally when it strikes and embeds in the bird. How far from the base of the tree do the bird and arrow land?

96. A dog of mass 10.0 kg is standing on a raft so that he is 20.0 m from shore. He walks 8.00 m along the raft towards shore and then halts. The raft has a mass of 40.0 kg, and we can assume there is no friction between the raft and the water. How far is the dog from shore when he stops?

97. Two men, of mass 100 kg each, stand on a cart of mass 300 kg. The cart can roll with negligible friction along a north-south track, and everything is initially at rest. One man runs towards the north and jumps off the cart at a speed of 5.0 m/s, relative to the cart. After he has jumped, the second man runs towards the south and jumps off the cart, again with a speed of 5.0 m/s relative to the cart. Calculate the speed and direction of the cart after both men have jumped off.

98. A 1000 kg plane is trying to make a forced landing on the deck of a 2000 kg barge at rest on the surface of a calm sea. The only frictional force to consider is between the plane’s wheels and the deck, and this braking force is constant and equal to one-quarter of the plane’s weight. What must the minimum length of the barge be, in order that the plane can stop safely on deck, if the plane touches down just at the rear end of the deck with a velocity of 50 m/s towards the front of the barge?

99. What velocity will a 40 kg child sitting on a 40 kg wagon acquire if pushed from rest by a force of 75 N for 2.0 s?

100. An 80 kg man standing at rest on a smooth, level ice surface throws a 200 g ball horizontally with a speed of 25 m/s, relative to Earth.
   (a) With what speed and in what direction does the man move?
   (b) If the man throws six such balls every 5.0 s, what is the average force acting on him?

101. How much work is done on an 8.0 kg wagon rolling along a flat sidewalk, if an applied force of 60 N opposite to its direction of motion brings it to rest in 2.0 s?

102. Calculate the work done by a horse that exerts an applied force of 100 N on a sleigh, if the harness makes an angle of 30° with the ground, and the sleigh moves 30 m across a flat, level ice surface.

103. How much work must be done to accelerate an 800 kg car from 15 m/s to 30 m/s?

104. Two small toys, one with a mass of 3.2 kg and the other with a velocity of 2.4 m/s, each have the same kinetic energy of 16 J. Determine the velocity of the first toy, and the mass of the second.
105. A bullet of mass 12 g strikes a stationary fixed block of wood at a speed of 400 m/s, penetrating to a depth of 3.0 cm. Calculate the average net force acting on the bullet while it is in the wood. Calculate the average force exerted on the wood by the bullet.

106. The graph shows the horizontal force on a 2.0 kg trolley as it moves 5.0 m along a straight, level, frictionless track, starting from rest. Determine its kinetic energy and velocity after each metre that it moves.

107. A 1.5 kg trolley moving to the right collides head-on with a 3.0 kg trolley initially at rest. The $F-x$ relationship for the collision is shown.

If the total kinetic energy of the two trolleys is 4.5 J when the separation distance between them is 0.05 m, what was the initial kinetic energy and velocity of the moving trolley?

108. A 1.0 kg cart moving at 2.5 m/s collides head-on with a 4.0 kg cart moving in the same direction at 0.50 m/s. The $F-x$ graph for the collision is shown.

The total kinetic energy at minimum separation is 2.0 J. What is the minimum separation of the two trolleys during the collision?

109. A 4.0 kg ball moving to the right at 5.0 m/s collides head-on with a 2.0 kg ball moving to the left at 4.0 m/s. If the collision is elastic, determine the direction and speed of each ball after the collision.
110. Sphere A, of mass 2.4 kg, moving in a straight line with velocity 10 m/s makes a head-on collision with sphere B, of mass 3.6 kg, which is initially at rest. The collision is cushioned by a perfectly elastic bumper.
   (a) What is the velocity of each sphere after the collision?
   (b) What percent of A’s kinetic energy is transferred to B by the collision?

111. A 2.0 kg trolley moving east at 3.0 m/s collides head-on with a 1.0 kg trolley moving west at 2.0 m/s. After the collision, the 2.0 kg trolley has a velocity of 1.0 m/s [E].
   (a) What is the final velocity of the 1.0 kg trolley?
   (b) Is the collision elastic or inelastic?

112. Two air track gliders of mass 300 g and 200 g are moving towards each other in opposite directions with speeds of 50 cm/s and 100 cm/s, respectively. Take the direction of the more massive glider as positive.
   (a) If the collision is elastic, find the velocity of each glider after the collision.
   (b) The most “inelastic” collision would occur if the two gliders stuck together on impact. If this were the case, find the velocity of the pair after the collision and the kinetic energy lost as a result of the collision.

113. A 6.0 kg trolley moving at 6.0 m/s [right] overtakes and collides with a 2.0 kg trolley moving at 2.0 m/s in the same direction on the same track. The collision is cushioned by a perfectly elastic bumper attached to one of the trolleys.
   (a) What is the speed and direction of each trolley after the collision?
   (b) What is the maximum amount of potential energy stored in the bumper during the collision?

114. A 4.0 kg object is at rest on a horizontal, frictionless surface when it is hit head-on by a 12.0 kg object moving forward at 0.80 m/s. The force separation graph for the collision is given.
   (a) Is the collision elastic or inelastic? Justify your answer.
   (b) Calculate the velocity of each object after the collision.
   (c) Calculate the velocity and kinetic energy of each object at minimum separation.
   (d) What is the minimum separation of the two? (Hint: Your answer to (c) enables you to find the loss in $E_k$ at minimum separation, which is equal to the potential energy stored in the bumper.)

115. Calculate the work done by the obvious agent in each case below:
(a) A locomotive exerts a constant forward force of \(5.4 \times 10^4\) N while pulling a train at a speed of 20 m/s for 1.0 h.
(b) A horse is towing a canal boat with a rope that makes an angle of 15° with the canal; the tension in the rope is 500 N and the canal is 120 m long.
(c) A 30 kg child climbs a flight of stairs 5.0 m high.
(d) A 12 kg suitcase is carried a horizontal distance of 30 m.
(e) An 8.0 kg sled experiences the horizontal force shown in the graph on a flat, frictionless sheet of ice.

\[ F(N) \]
\[ 4 \]
\[ 2 \]
\[ 1 \]
\[ 2 \]
\[ 3 \]
\[ 4 \]
\[ 5 \]
\[ 6 \]
\[ 7 \]
\[ 8 \]
\[ d(m) \]

116. A 40 kg wagon is moving with a constant horizontal velocity of 10 m/s.
(a) How much work must be done to double this velocity?
(b) How much work must be done to halve the original velocity?

117. From what height must a 1500 kg piledriver fall, to drive a pile 0.50 m into the ground against an average opposing force of \(3.5 \times 10^5\) N? (Ignore the weight of the piledriver over the 0.50 m path.)

118. A force that increases gradually from 0 to 100 N at a rate of 10 N/s does 12 500 J of work on an object, accelerating it from rest to a velocity of 50 m/s on a flat, frictionless surface.
(a) What is the mass of the object?
(b) What constant force would have given the object the same final velocity if the force had acted over a distance of 5.0 m?

119. A 4.0 kg rock moving at 20 m/s has the same momentum as a 10.0 kg rock.
(a) What is the velocity of the 10.0 kg rock?
(b) What is the kinetic energy of each?

120. A pellet of mass 5.0 g is fired from a heavy gun whose barrel is 100 cm long. The force on the pellet while it is in the barrel is given by the graph. What is the muzzle velocity of the pellet?

\[ F(N) \]
\[ 1.2 \times 10^2 \]
\[ 0.6 \times 10^2 \]
\[ 0 \]
\[ 0.2 \]
\[ 0.4 \]
\[ 0.6 \]
\[ 0.8 \]
\[ 1.0 \]
\[ d(m) \]

121. A ball of mass 0.80 kg moving initially at 8.0 m/s has a head-on collision with a 0.40 kg ball that is at rest. If the collision is perfectly elastic, what is the velocity of each ball after the collision?

122. A truck of mass 3000 kg, moving at 5.0 m/s on a level, icy road, bumps into the rear of a car moving at 2.0 m/s in the same direction. After the impact the truck has a velocity of 3.0 m/s and the car a velocity of 6.0 m/s, both forward.
(a) What is the mass of the car?
(b) Calculate the total kinetic energy before and after the collision.
(c) Was the collision elastic?
123. A 1.0 kg magnetized air puck moving across a level table at 0.24 m/s approaches head-on a stationary, similarly magnetized air puck of mass 0.50 kg. If the “magnetic collision” is repulsive and perfectly elastic, determine:
   (a) the velocity of each puck after the collision
   (b) the velocity of both pucks at minimum separation
   (c) the total kinetic energy at minimum separation
   (d) the maximum potential energy stored in the magnetic force field during the collision

124. On a frictionless air track, a 0.30 kg glider moving at 0.40 m/s to the right collides with a 0.80 kg glider moving at 0.15 m/s to the left. The collision is cushioned by a bumper made of perfectly elastic spring steel.
   (a) What is the velocity of each glider after the collision?
   (b) What is the minimum amount of total kinetic energy during the collision?
   (c) Where is the missing energy?

125. Two identical billiard balls are at rest on a level, frictionless surface, just touching each other at one common point, as shown. A third, identical ball, the cue ball, is approaching along the common tangent with a constant speed of 20 m/s, as shown. Assuming a completely elastic collision, with no spin on any of the balls, and making (reasonable) assumptions about symmetry, calculate the velocity of the cue ball after the collision.

126. A bullet of mass 4.0 g, moving horizontally with a velocity of 500 m/s, strikes a wooden block of mass 2.0 kg, initially at rest on a rough, horizontal surface. The bullet passes through the block in a negligible time interval, emerging with a velocity of 100 m/s and causing the block to slide 40 cm along the surface before coming to rest.
   (a) With what velocity does the wood block move just after the bullet exits?
   (b) What is the maximum kinetic energy of the block?
   (c) What is the average frictional force stopping the block?
   (d) What is the decrease in kinetic energy of the bullet?
   (e) Explain why the decrease in $E_k$ of the bullet and the maximum $E_k$ of the block are not equal. What happened to this difference in energy?

127. A massless spring is compressed between blocks of mass $m$ and $5m$ on a smooth, horizontal table. When the system is released, the energy of the spring is shared between the blocks. What fraction of the total energy does the smaller block acquire?

128. A 2.5 kg mass, at rest, is approached head-on by a 5.0 kg mass moving at 0.60 m/s. The force-separation graph for the ensuing collision is given.

   a. What is the total kinetic energy before the collision? After?
b. What is the velocity of each mass at minimum separation?
c. What is the total kinetic energy at minimum separation?
d. How much energy is stored at minimum separation?
e. What is the minimum separation distance?
f. What is the magnitude of the force acting on each mass at minimum separation?

129. Calculate the work done in lifting a 12 kg crate from the floor to a platform 3.0 m above floor level.

130. A block of mass 0.50 kg is placed on a level, frictionless surface, in contact with a spring bumper, of force constant 100 N/m, that has been compressed by an amount 0.30 m. The spring, whose other end is fixed, is then released. What is the speed of the block at the instant when the spring is still compressed by 0.10 m?

131. A cart of mass 2.0 kg is moving to the right along a smooth, horizontal track at 3.0 m/s. A Hooke’s Law spring, of force constant 1200 N/m and normal length 25 cm, is attached to its front. It collides “head-on” with a 4.0 kg cart, initially at rest.
   (a) What is the velocity of each cart after the collision?
   (b) What is the velocity of each cart at minimum separation?
   (c) What is the change in total kinetic energy at minimum separation?
   (d) What is the compression of the spring at minimum separation?
   (e) What is the minimum separation of the two carts?

132. Calculate the force constant of a spring that, when compressed 10 cm between two stationary 2.5 kg carts and released, causes each cart to move off with a velocity of 3.0 m/s.

133. Two carts, of mass 4.5 kg and 1.0 kg, are moving towards each other along the same straight, level track at 2.0 m/s to the right and 4.0 m/s to the left, respectively. Their collision is cushioned by a linear elastic spring between them.
   (a) What is the total energy of the system before the collision?
   (b) At minimum separation, what is the velocity of each cart?
   (c) Calculate the total kinetic energy at minimum separation.
   (d) If the force constant of the spring is 900 N/m, what is its maximum compression during the collision?

134. A glider of mass 50 g is moving to the left along an air track at a speed of 4.0 cm/s when it collides with a second glider of mass 30 g moving to the right with a speed of 20 cm/s. The collision is perfectly elastic, cushioned by a steel-loop spring that obeys Hooke’s Law.
   (a) Determine the velocity of each glider at that point during the collision when their separation distance is a minimum.
   (b) Determine the amount of elastic potential energy stored in the spring at that instant.
   (c) If the maximum deformation of the spring during the collision is 1.5 cm, what is its spring constant?

135. A ball of mass 0.25 kg is thrown vertically upward from the roof of a building 18 m high with a speed of 16 m/s, and just misses the building on the way down, as shown.
   (a) To what vertical height above Earth does the ball rise?
   (b) With what vertical velocity does the ball hit the ground?
136. A 1000 kg roller coaster, with its passengers, starts from rest at point A on a frictionless track whose profile is shown in the diagram.
(a) What is its maximum speed?
(b) With what speed does the roller coaster arrive at point E?
(c) What constant braking force would have to be applied to the roller coaster at point E, to bring it to rest in a horizontal distance of 5.0 m?

137. A Hooke’s Law spring is compressed 10 cm by an applied force of 50 N. This compressed spring is then used to project a 20 g marble straight up into the air. To what maximum height does the marble rise?

138. A 70 kg diver jumps from a 12 m tower, with no initial velocity.
(a) With what velocity does the diver hit the water?
(b) What would his impact velocity be if, in jumping from the tower, he gave himself an upward initial velocity of 5.0 m/s? (Hint: It is a waste of time to calculate his maximum height reached).

139. What upward velocity must an Olympic high jumper impart to herself in order to clear the 2.0 m height of the bar? (Hint: Assume that she must raise her centre of mass to just above the bar, and that her centre of mass is 0.85 m above ground level when she begins to jump.)

140. A 2.0 kg mass is placed against a spring of force constant 800 N/m, which has been compressed 0.22 m, as illustrated. The spring is released, and the object moves along the horizontal, frictionless surface and up the slope.

Calculate:
(a) the maximum elastic potential energy of the spring
(b) the maximum velocity of the mass
(c) the maximum vertical height of the mass, up the slope
141. A ball bearing of mass 50 g is sitting on a vertical spring whose force constant is 120 N/m. By how much must the spring be compressed so that, when released, the ball rises to a maximum height of 3.1 m above its release position?

142. With what initial velocity must an object be projected vertically upward from the surface of Earth, in order to rise to a height equal to Earth’s radius? (Neglect air resistance and the rotation of Earth.)

143. Calculate the change in gravitational potential energy for a 1.0 kg mass lifted 100 km above the Earth’s surface. What percentage error would have been made by using the equation $\mathcal{E}_g = mg\Delta h$ and the value of $g$ at Earth’s surface ($m_E = 5.98 \times 10^{24}$ kg, $r_E = 6.37 \times 10^6$ m)? What does this tell you about the need for the more exact treatment in most normal Earth-bound problems?

144. A 500 kg communications satellite is to be placed in a synchronous orbit around Earth.
   (a) What is the radius of its circular orbit?
   (b) What is the gravitational potential energy of the satellite when it is attached to its launch rocket, at rest on Earth’s surface?
   (c) What is the total energy of the satellite when synchronous in orbit?
   (d) How much work must the launch rocket do on the satellite to place it in orbit?
   (e) Once in orbit, how much additional energy must the satellite receive, in order to escape from Earth’s potential well?

145. (a) What is the total amount of energy needed to place a 2000 kg satellite in circular Earth orbit, at an altitude of 500 km?
    (b) How much additional energy would have to be supplied to the satellite, once it was in orbit, to allow it to escape from Earth’s gravitational field?

146. The spring in a toy gun has a force constant of 500 N/m. It is compressed 5.0 cm, and a ball of mass 10 g is placed next to it in the barrel.
   (a) What is the ball’s maximum velocity, when the trigger releases the spring? Assume that there is no friction, and that the gun barrel is level.
   (b) Determine the muzzle velocity if an average retarding force of 0.80 N acts on the ball, and the barrel is 0.25 m long.

147. A rifle bullet of mass 10.0 g strikes and becomes embedded in a wooden block of mass 490 g, which is at rest on a horizontal, frictionless surface and is attached to a spring bumper, as shown.

The impact compresses the spring, whose force constant is 100 N/m, by 20 cm.
   (a) What is the maximum potential energy of the spring?
   (b) Determine the velocity with which the block and bullet first begin to move.
   (c) What was the initial velocity of the bullet?
   (d) What was the initial kinetic energy of the bullet?
   (e) Explain any difference between (a) and (d).

148. A 2.4 kg dynamics cart is moving to the right at 1.5 m/s, with a linear elastic spring attached to its front end, when it collides head-on with a stationary cart of mass 3.6 kg.
   (a) Calculate the total energy of the system before the collision.
   (b) What is the velocity of each cart at minimum separation?
   (c) Calculate the change in total kinetic energy of the system at minimum separation.
(d) If the maximum compression of the spring during the collision is 12 cm, what is its force constant?

149. Two trolleys, of mass 1.2 kg and 4.8 kg, are at rest with a compressed spring between them, held that way by a string tied around both. When the string is cut, the trolleys spring apart. If the force constant of the spring is 2400 N/m, by how much must it have been compressed in order that the 4.8 kg cart move off at 2.0 m/s?

150. A 3.0 kg ball is dropped from a height of 0.80 m onto a vertical spring of force constant 1200 N/m. What is the maximum compression of the spring?

151. A 60 kg person jumps from a platform onto a trampoline 10 m below, stretching it 1.0 m from its initial position. Assuming that the trampoline behaves like a simple elastic spring, how much will it stretch if the same person jumps from a height of 20 m?

152. A linear elastic spring is 15.0 cm long. When the upper end is held in the hand and a 500 g mass is suspended from the lower end, its length becomes 22.0 cm. If the hand is now jerked quickly upward, the spring first extends to a length of 28.5 cm, then the mass starts to move up. The hand is then held still.
   (a) Calculate the acceleration with which the mass first begins to move.
   (b) Calculate the speed of the mass when the length of the spring becomes 22.0 cm again.
   (c) Calculate the length of the spring when the mass comes to rest at its highest point.

153. A block of mass 1.0 kg, at rest on a horizontal table as shown, is attached to two rigid supports by springs A and B. A force of 10 N stretches spring A alone by 0.25 m while a force of 2.5 N extends spring B alone by the same amount. Initially the block is at rest between the unstretched springs; then it is pushed to the side a distance of 0.50 m by a variable horizontal force \( F \), compressing one spring and extending the other.

(a) What is the total work done by the force \( F \)? (The block is held at rest.)
(b) If the block is then released, with what velocity does it move through its original equilibrium position?
(c) What would be the spring constant of a single spring that would duplicate A and B?

154. A 1.0 kg lead sphere is suspended from the ceiling by a wire 5.0 m long. The ball is pulled sideways and up, until the wire is horizontal, and then released. Find
   (a) the maximum velocity acquired by the ball
   (b) the tension in the wire at the lowest point in the swing

155. A toy cart of mass 5.0 kg is projected up a ramp inclined at 30° to the horizontal, with an initial velocity of 6.0 m/s. If the frictional force opposing its motion is 4.0 N, find the distance it travels before stopping, and its increase in gravitational potential energy at that point.

156. A new amusement park ride is shown in the diagram.
The mass of the small car, including its contents, is 400 kg, and the radius of the loop is 10 m. Assume that the track is frictionless and that no mechanism holds the car in contact with the track.

a. In order that the car stay on the track at B, what minimum velocity must it have at that point?

b. From what minimum height, \( h \), must the car start from rest in order to do this?

c. Is the mass of the car a factor in determining \( h \)? Explain.

157. Two boxes are connected by a smooth rope over a light, frictionless, fixed pulley, as shown. Use the principle of conservation of energy to determine the velocity with which the 15.0 kg box strikes the floor, after the system is released from rest.

158. A 1.0 kg tetherball is suspended by a string 0.80 m long from a nail at point X, 0.50 m vertically above a horizontal rod at Y. When the ball is released from Z, on the same horizontal level as Y, it swings down, and the string catches on Y.

(a) What is the initial gravitational potential energy of the 1.0 kg tetherball relative to point X?

(b) What is its initial potential energy relative to point Y?

(c) Determine the location of points A, B, and C, where the mass has, respectively:
   (i) maximum kinetic energy
   (ii) maximum velocity
   (iii) maximum potential energy with respect to Y after it is released

Calculate the value of each of the quantities in (i), (ii), and (iii).

(d) How high above Y must the tetherball be released so that it will swing completely around a circle of radius 0.30 m about point Y?

159. (a) Isaac Newton was not inspired by an apple falling on his head. Actually, he was lying down and the apple struck his stomach. It then bounced straight back up, having lost 10% of its kinetic energy in the collision. How high did it rise on the first bounce if it had originally dropped from a branch 1.0 m above Isaac’s stomach?

(b) The same apple continued to bounce, losing the same fraction of its kinetic energy each time. If you ignore the slight deformation of Isaac’s stomach during each bounce, what was the total distance travelled by the apple from the time it left the tree until it eventually came to rest on Isaac’s stomach?

160. The space shuttle ejects a 1200 kg booster tank so that the tank is momentarily at rest at an altitude of 2000 km. Neglecting atmospheric effects, determine

(a) how much work is done on the booster by the force of gravity in returning it to Earth’s surface

(b) the velocity with which it strikes the surface of Earth

161. An artificial Earth satellite, of mass \( 2.00 \times 10^3 \) kg, has an elliptical orbit, with a mean altitude of 400 km.

(a) What is its mean value of gravitational potential energy while in orbit?

(b) What is its mean value of orbital kinetic energy?

(c) What is its total energy while in orbit?

(d) If its perigee is 280 km, what is its orbital velocity at perigee?
162. A 500 kg satellite is in circular orbit 200 km above Earth’s surface. Calculate:
(a) the gravitational potential energy of the satellite
(b) the kinetic energy of the satellite
(c) its binding energy
(d) the percentage increase in launching energy required to make it escape from Earth

163. (a) Calculate the escape velocity from our solar system (i.e., from the surface of the Sun, whose mass is \(1.98 \times 10^{30}\) kg, and whose radius is \(6.96 \times 10^8\) m).
(b) What velocity would an object leaving Earth need, to escape from our solar system?

164. The mass of the Moon is approximately \(6.7 \times 10^{22}\) kg, and its radius is \(1.6 \times 10^6\) m.
(a) With what velocity must an object be projected from the Moon’s surface in order to rise to an altitude equal to the Moon’s radius?
(b) If a woman can raise her centre of gravity 2.0 m vertically in a high jump at Earth’s surface, how high could she jump with the same muscular effort on the Moon’s surface?

165. Realizing that he could not drive up a 30°, ice-covered hill because there was no friction, Sir Isaac Newton had stopped his cart, of total mass 500 kg, at the bottom. He was struck in the rear by a London stage coach, of total mass 1500 kg, travelling at 20 m/s. The two vehicles stuck together, with nothing breaking loose, and slid up the hill in a straight line. How far up the slope did the wreckage get before coming to rest?

166. A very light basket hangs from the limb of a tree by a long spring. The limb extends out over a pond, and the spring holds the basket 3.0 m above the surface of the pond. Three girls of equal mass carefully lower themselves into the basket, one after the other, causing the spring to stretch 1.0 m for each additional girl, so that with all three aboard, the basket just touches the water. The girls then jump into the water, and the basket returns to its original position. Once back on shore, one of the girls climbs to a higher limb of the tree and steps off, landing in the basket and causing the spring to stretch until the basket just touches the water’s surface again, for an instant. From what height above the water’s surface did the girl step from the higher limb?

167. The force-compression graph of a hypothetical spring is shown below.

![Force-compression graph](image)

(a) How much work is done in compressing the spring 0.60 m?
(b) What is the potential energy stored in the spring when compressed this amount?
(c) A 5.0 kg toy car is placed on a level, frictionless surface against the spring when compressed 0.60 m, and the spring is released. The other end of the spring is fixed. Determine the velocity of the car when it has moved a distance of 0.20 m.

168. A rocketship, of mass \(1.00 \times 10^4\) kg is located \(1.00 \times 10^{10}\) m from the centre of Earth.
(a) Determine its gravitational potential energy at this point, considering only Earth.
(b) How much kinetic energy must it have, at this point, to be capable of escaping from Earth’s gravitational field?
(c) What is its escape velocity from Earth, at this point?
Momentum & Energy Extra Study Questions
Answer Section

SHORT ANSWER

1. ANS:  
\[ \vec{p} = m \vec{v} \]
\[ = \left( 1000 \text{ kg} \right) \left( 15 \text{ m/s \ [E]} \right) \]
\[ = 1.5 \times 10^4 \text{ kg} \cdot \text{m/s \ [E]} \]

REF: K/U OBJ: 5.1 LOC: EM1.01 KEY: FOP 8.1, p.290
MSC: SP

2. ANS:
   (a) \[ \vec{p} = m \vec{v} \]
   \[ = \left( 0.50 \text{ kg} \right) \left( 30 \text{ m/s \ [up]} \right) \]
   \[ = 15 \text{ kg} \cdot \text{m/s \ [up]} \]

   (b) \[ \vec{p} = m \vec{v} \]
   \[ = \left( 2000 \text{ kg} \right) \left( 10 \text{ m/s \ [S]} \right) \]
   \[ = 2.0 \times 10^4 \text{ kg} \cdot \text{m/s \ [S]} \]

   (c) \[ \vec{p} = m \vec{v} \]
   \[ = \left( 9.1 \times 10^{-31} \text{ kg} \right) \left( 1.0 \times 10^7 \text{ m/s} \right) \]
   \[ = 9.1 \times 10^{-24} \text{ kg} \cdot \text{m/s \ [forward]} \]

   (d) \[ \vec{p} = m \vec{v} \]
   \[ = \left( 6.0 \times 10^{-24} \text{ kg} \right) \left( 3.0 \times 10^4 \text{ m/s} \right) \]
   \[ = 1.8 \times 10^9 \text{ kg} \cdot \text{m/s \ [forward]} \]

REF: K/U OBJ: 5.1 LOC: EM1.01 KEY: FOP 8.1, p.290
MSC: P

3. ANS:
\[ \vec{v} = \frac{\vec{p}}{m} \]
\[ \begin{align*}
\text{22 \text{ kg} \cdot \text{m/s} \ \text{[forward]}} \\
\text{7.3 \text{ kg}} \\
\text{= 3.0 \text{ m/s} \ \text{[forward]}}
\end{align*} \]

REF: K/U OBJ: 5.1 LOC: EM1.01 KEY: FOP 8.1, p.291
MSC: P

4. ANS:
\[ m = \frac{\vec{p}}{\vec{v}} \]
\[ \begin{align*}
\text{4.5 \text{ kg} \cdot \text{m/s} \ \text{[forward]}} \\
\text{900 \text{ m/s} \ \text{[forward]}} \\
\text{= 5.0 \times 10^{-3} \text{ kg, or 5.0 g}}
\end{align*} \]

REF: K/U OBJ: 5.1 LOC: EM1.01 KEY: FOP 8.1, p.291
MSC: P

5. ANS:
\[ \text{impulse} = \vec{F}_{\text{avg}} \Delta t \]
\[ \begin{align*}
\text{(500 \text{ N})(0.0050 \text{ s})} \\
\text{= 2.5 \text{ N} \cdot \text{s} \ \text{[forward]}}
\end{align*} \]

REF: K/U OBJ: 5.1 LOC: EM1.01 KEY: FOP 8.2, p.294
MSC: SP

6. ANS:
\[ \text{impulse} = \text{area under } \vec{F} \text{ versus } t \text{ graph during } \Delta t \]
\[ = \text{area triangle I + area rectangle II + area triangle III} \]
\[ = \frac{1}{2} (0.02 \text{ s})(60 \text{ N}) + (0.02 \text{ s})(60 \text{ N}) + \frac{1}{2} (0.02 \text{ s})(60 \text{ N}) \]
\[ = 2.4 \text{ N} \cdot \text{s} \ \text{[forward]} \]

REF: K/U OBJ: 5.1 LOC: EM1.01 KEY: FOP 8.2, p.294
MSC: SP

7. ANS:
\[ \text{(a) impulse} = \vec{F} \Delta t \]
\[ \begin{align*}
\text{(25 \text{ N} \ [E])(3.2 \text{ s})} \\
\text{= 80 \text{ N} \cdot \text{s} \ [E]}
\end{align*} \]
(b) impulse = $\vec{F}\Delta t$
\[= (120 \text{ N})(0.05 \text{ s})\]
\[= 6 \text{ N} \cdot \text{s} \text{ [in the direction of the shot]}\]

(c) impulse = $\vec{F}\Delta t$
\[= m \vec{g}\Delta t\]
\[= (12 \text{ kg})(9.8 \text{ N/kg [down]})(3.0 \text{ s})\]
\[= 3.5 \times 10^2 \text{ N} \cdot \text{s} \text{ [down]}\]

(d) impulse = area under $F\cdot t$ graph
\[= \frac{1}{2} (0.5 \text{ s})(4 \text{ N})\]
\[= 1 \text{ N} \cdot \text{s} \text{ [in the direction opposite the ball's initial motion]}\]

(e) impulse = area under $F\cdot t$ graph
\[\approx \frac{1}{2} (0.3 \text{ s})(10 \text{ N})\]
\[\approx 1.5 \text{ N} \cdot \text{s} \text{ [in the direction opposite the car's initial motion]}\]

REF: K/U OBJ: 5.1 LOC: EM1.01 KEY: FOP 8.2, p.295
MSC: P

8. ANS:
\[\vec{F}\Delta t = m\vec{v}_2 - m\vec{v}_1\]
\[\vec{F} = \frac{m\vec{v}_2 - m\vec{v}_1}{\Delta t}\]
\[= \frac{(1000 \text{ kg})(0 \text{ m/s}) - (1000 \text{ kg})(22 \text{ m/s})}{1.5 \text{ s}}\]
\[= \frac{-22000 \text{ kg} \cdot \text{m/s}}{1.5 \text{ s}}\]
\[= -1.5 \times 10^4 \text{ N} \text{ [in the direction of motion]}\]

REF: K/U OBJ: 5.1 LOC: EM1.01 KEY: FOP 8.2, p.296
MSC: SP

9. ANS:
\( \Delta \vec{p} = \vec{p}_2 - \vec{p}_1 \)
\[
= m \vec{v}_2 - m \vec{v}_1 \\
= (0.200 \text{ kg})(-2.0 \text{ m/s}) - (0.200 \text{ kg})(+2.0 \text{ m/s}) \\
= -0.80 \text{ kg} \cdot \text{ m/s}[\text{R}]
\]

(b) impulse \( = \Delta \vec{p} \)
\[
= -0.80 \text{ kg} \cdot \text{ m/s} [\text{R}] \\
= 0.80 \text{ N} \cdot \text{s}[\text{L}]
\]

10. ANS: 
(a) \( \Delta \vec{p} = \vec{p}_2 - \vec{p}_1 \)
\[
= m \vec{v}_2 - m \vec{v}_1 \\
= (0.20 \text{ kg})(10 \text{ m/s}) - (0.20 \text{ kg})(18 \text{ m/s}) \\
= -1.6 \text{ kg} \cdot \text{ m/s} \text{ [forward]} \\
\]
(b) impulse \( = \Delta \vec{p} \)
\[
= -1.6 \text{ kg} \cdot \text{ m/s} \text{ [forward]} \\
= 1.6 \text{ N} \cdot \text{s} \text{ [backward]} \\
\]
(c) impulse \( = \vec{F} \cdot \Delta t \)
\[
\therefore \vec{F} = \frac{\text{impulse}}{\Delta t} \\
= \frac{1.6 \text{ N} \cdot \text{s} \text{ [backward]}}{2.5 \text{ s}} \\
= 0.64 \text{ N} \text{ [backward]} \\
\]

11. ANS:
12. ANS:

\[ m_1 \vec{v}_1 + m_2 \vec{v}_2 = (m_1 + m_2) \vec{v}_{12} \]

\[
(5000 \text{ kg})(5.2 \text{ m/s}) + (8000 \text{ kg})(0 \text{ m/s}) = (13000 \text{ kg}) \vec{v}_{12}
\]

\[ \vec{v}_{12} = \frac{26000 \text{ kg} \cdot \text{m/s}}{13000 \text{ kg}} \]

\[ \vec{v}_{12} = 2.0 \text{ m/s [forward]} \]

13. ANS:

\[ m_1 \vec{v}_1 + m_2 \vec{v}_2 = (m_1 + m_2) \vec{v}_{12} \]

\[
(75 \text{ kg})(3.0 \text{ m/s}) + (10 \text{ kg})(2.0 \text{ m/s}) = (85 \text{ kg}) \vec{v}_{12}
\]

\[ \vec{v}_{12} = \frac{245 \text{ kg} \cdot \text{m/s}}{85 \text{ kg}} \]

\[ \vec{v}_{12} = 2.9 \text{ m/s [forward]} \]

14. ANS:

\[ m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 \vec{v}_1' + m_2 \vec{v}_2' \]

\[
(100 \text{ g})(200 \text{ cm/s}) + (400 \text{ g})(0) = (100 \text{ g})(-120 \text{ cm/s}) + (400 \text{ g}) \vec{v}_2'
\]

\[ \vec{v}_2' = \frac{20000 \text{ g} \cdot \text{cm/s} + 12000 \text{ g} \cdot \text{cm/s}}{400 \text{ g}} \]

\[ \vec{v}_2' = 80 \text{ cm/s [forward]} \]
15. ANS:
Since the masses of both balls are equal, $\vec{p}'$'s are proportional to $\vec{v}'$'s on vector diagrams.
$\vec{p}_1' = \vec{p}'_1 = 10M \text{ cm/s} \ [R]$.

Scale: 1 cm = 2M cm/s

$\vec{p}_1' = 4.35 \text{ cm}$
$\therefore \quad v_1' = 8.7M \text{ cm/s}$
16. ANS:

\[ p = mv \]

\[ v = \frac{p}{m} \]

\[ = \frac{3.0 \times 10^4 \text{ kg} \cdot \text{m/s}}{1800 \text{ kg}} \]

\[ = 17 \text{ m/s} \]

REF: K/U OBJ: 5.1 LOC: EM1.01 KEY: FOP 8.8, p.327

17. ANS:

(a) \[ p_2 = mv_2 \]

\[ = m \left( v_1 + a \Delta t \right) \]

\[ = \left( 1.5 \times 10^2 \text{ kg} \right) \left[ 0 + \left( \frac{4.0 \text{ m/s}^2}{(6.0 \text{ s})} \right) \right] \]

\[ = 3.6 \times 10^4 \text{ kg} \cdot \text{m/s} \]

(b) impulse = \[ \Delta p \]

\[ = p_2 - p_1 \]

\[ = 3.6 \times 10^4 \text{ kg} \cdot \text{m/s} - 0 \text{ kg} \cdot \text{m/s} \]

\[ = 3.6 \times 10^4 \text{ kg} \cdot \text{m/s} \]

\[ = 3.6 \times 10^4 \text{ N} \cdot \text{s} \]

REF: K/U OBJ: 5.1 LOC: EM1.01 KEY: FOP 8.8, p.327

18. ANS:

(a) impulse = \[ F \Delta t \]

\[ = (350 \text{ N})(0.12 \text{ s}) \]

\[ = 42 \text{ N} \cdot \text{s} \]

(b) \[ \Delta p = \text{impulse} \]

\[ = 42 \text{ kg} \cdot \text{m/s} \]

REF: K/U OBJ: 5.1 LOC: EM1.01 KEY: FOP 8.8, p.327

19. ANS:
(a) impulse = $\Delta p$

$$= p_2 - p_1 \quad \text{(but } p_1 = 0 \text{ since } v_1 = 0)$$

$$= m v_2$$

$$= (5.0 \text{ kg})(200 \text{ m/s})$$

$$= 1.0 \times 10^3 \text{ kg} \cdot \text{m/s}$$

$$= 1.0 \times 10^3 \text{ N} \cdot \text{s}$$

(b) impulse = $F \Delta t$

$$\therefore \Delta t = \frac{\text{impulse}}{F}$$

$$= \frac{1.0 \times 10^3 \text{ N} \cdot \text{s}}{5.0 \times 10^4 \text{ N}}$$

$$= 2.0 \times 10^{-2} \text{ s}$$

REF: K/U OBJ: 5.1 LOC: EM1.01 KEY: FOP 8.8, p.327

MSC: P

20. ANS:

(a) $v_2^2 = v_1^2 + 2a \Delta d$

$$= 0 \text{ m/s} + 2 \left( -9.8 \text{ m/s}^2 \right)(-4.5 \text{ m})$$

$$= +38.2 \text{ (m/s)}^2$$

$$\therefore v_2 = -9.4 \text{ m/s} \text{ since down is negative}$$

(b) impulse = $\Delta p$

$$= p_2 - p_1$$

$$= 0 \text{ N} \cdot \text{s} - m v_1$$

$$= - (80 \text{ kg})(-9.4 \text{ m/s})$$

$$= 7.5 \times 10^2 \text{ N} \cdot \text{s}$$
21. ANS:
For a rocket,
\[ F = \frac{\Delta m_g \nu_g}{\Delta t} \]
\[ \frac{\Delta m_g}{\Delta t} = \frac{F}{\nu_g} \]
\[ = \frac{(4.0 \text{ N})}{30 \text{ m/s}} \]
\[ = 1.3 \times 10^{-1} \text{ kg/s, or 0.13 kg/s} \]

22. ANS:
Momentum is conserved in the interaction.
\[ m_1 \nu_1 + m_2 \nu_2 = (m_1 + m_2) \nu'_{12} \]
\[ \nu'_{12} = \frac{m_1 \nu_1 + m_2 \nu_2}{m_1 + m_2} \]
\[ = \frac{\left( 1.2 \times 10^4 \text{ kg}(25 \text{ m/s}) + (3000 \text{ kg})(0 \text{ m/s}) \right)}{1.2 \times 10^4 \text{ kg} + 0.3 \times 10^4 \text{ kg}} \]
\[ = 20 \text{ m/s} \]
23. ANS:
Momentum is conserved in the interaction,
\[ \vec{P}_{\text{total}} = \vec{P}_{\text{total}} \]
\[ m_1 \vec{v}_1 + m_2 \vec{v}_2 = (m_1 + m_2) \vec{v}_{12} \]
\[ (45 \text{ kg})(4.0 \text{ m/s}) + (15 \text{ kg})(0) = (15 \text{ kg} + 45 \text{ kg}) \vec{v}_{12} \]
\[ \vec{v}_{12} = \frac{180 \text{ kg} \cdot \text{m/s}}{60 \text{ kg}} \]
\[ = 3.0 \text{ m/s} \]

REF: K/U OBJ: 5.2 LOC: EM1.02 KEY: FOP 8.8, p.328
MSC: P

24. ANS:
Momentum is conserved in the collision,
\[ \vec{P}_{\text{total}} = \vec{P}_{\text{total}} \]
\[ m_1 \vec{v}_1 + m_2 \vec{v}_2 = (m_1 + m_2) \vec{v} \]
\[ \left(1.2 \times 10^3 \text{ kg}\right)(20 \text{ m/s}) + \left(1.0 \times 10^3 \text{ kg}\right)(0 \text{ m/s}) = \left(2.2 \times 10^3 \text{ kg}\right) \vec{v} \]
\[ \vec{v} = \frac{2.4 \times 10^4 \text{ kg} \cdot \text{m/s}}{2.2 \times 10^3 \text{ kg}} \]
\[ = 1.1 \times 10^1 \text{ m/s}, \text{ or } 11 \text{ m/s} \]

REF: K/U OBJ: 5.2 LOC: EM1.02 KEY: FOP 8.8, p.329
MSC: P

25. ANS:
Momentum is conserved in the collision,
\[ \vec{P}_{\text{total}} = \vec{P}_{\text{total}} \]
\[ m_1 \vec{v}_1 + m_2 \vec{v}_2 = (m_1 + m_2) \vec{v}_{12} \]
\[ m_1(40 \text{ m/s}) + (0.400 \text{ kg})(0 \text{ m/s}) = (m_1 + 0.400 \text{ kg})(10 \text{ m/s}) \]
\[ m_1 = \frac{4.00 \text{ kg} \cdot \text{m/s}}{30 \text{ m/s}} \]
\[ = 0.13 \text{ kg} \]

REF: K/U OBJ: 5.2 LOC: EM1.02 KEY: FOP 8.8, p.329
MSC: P

26. ANS:
Momentum is conserved in the interaction:
\[ \vec{P}_{\text{total}} = \vec{P}_{\text{total}}, \text{ where } \vec{P}_{\text{total}} = 0 \]
\[ 0 = m_1 \vec{v}_1 + m_2 \vec{v}_2 \]
\[ 0 = (8.0 \text{ kg})(600 \text{ m/s}) + (500 \text{ kg})(\vec{v}_2) \]
\[ \vec{v}_2 = -\frac{4800 \text{ kg \cdot m/s}}{500 \text{ kg}} \]
\[ = -9.6 \text{ m/s} \]

27. ANS:
Momentum is conserved in this collision.
\[ \vec{P}_{\text{total}} = \vec{P}_{\text{total}} \]
\[ m_1 \vec{v}_1 + m_2 \vec{v}_2 = (m_1 + m_2) \vec{v}_{12} \]
\[ (0.030 \text{ kg})(4.0 \text{ m/s}) + (0.080 \text{ kg})(-1.5 \text{ m/s}) = (0.110 \text{ kg})\vec{v}_{12} \]
\[ \vec{v}_{12} = \frac{0.120 \text{ kg \cdot m/s} - 0.120 \text{ kg \cdot m/s}}{0.110 \text{ kg}} \]
\[ = 0 \text{ m/s} \]

28. ANS:
(a) Momentum is conserved when he pushes off:
\[ \vec{P}_{\text{total}} = \vec{P}_{\text{total}}, \text{ where } \vec{P}_{\text{total}} = 0 \]
\[ 0 = m_1 \vec{v}_1 + m_2 \vec{v}_2 \]
\[ 0 = (125 \text{ kg})(2.0 \text{ m/s}) + (2500 \text{ kg})\vec{v}_2 \]
\[ \vec{v}_2 = \frac{-250 \text{ kg \cdot m/s}}{2500 \text{ kg}} \]
\[ = -0.10 \text{ m/s} \]

(b) After push-off, relative velocity of astronaut and spaceship is
\[ \nu = 2.0 \text{ m/s} + 0.10 \text{ m/s} \]
\[ = 2.10 \text{ m/s} \]
\[ \Delta t = \frac{\Delta d}{v} \]
\[ = \frac{25 \text{ m}}{2.10 \text{ m/s}} \]
\[ = 11.9 \text{ s, or 12 s} \]

(c) The centre of mass of the system remains at rest, since momentum cannot change from zero. It was close to the capsule centre, but the capsule moved.
\[ \Delta d = v \Delta t \]
\[ = (0.10 \text{ m/s})(11.9 \text{ s}) \]
\[ = 1.19 \text{ m or 1.2 m} \]

REF: K/U, MC  OBJ: 5.2  LOC: EM1.02  KEY: FOP 8.8, p.329
MSC: P

29. ANS:
Momentum in conserved in the fission:
\[ \vec{p}_{\text{total}} = \vec{p}_{\text{total}} = 0 \]
\[ 0 = m_1 \vec{v}_1 + m_2 \vec{v}_2 \]
\[ 0 = (1)(1000 \text{ m/s}) + (50)(-2.3 \times 10^4 \text{ m/s}) \]
\[ \therefore v_1 = 1.38 \times 10^6 \text{ m/s, or 1.4 \times 10^6 m/s} \]

REF: K/U  OBJ: 5.2  LOC: EM1.02  KEY: FOP 8.8, p.329
MSC: P

30. ANS:
(a) Momentum is conserved in the collision:
\[ \vec{p}_{\text{total}} = \vec{p}_{\text{total}} \]
\[ m_1 \vec{v}_1 + m_2 \vec{v}_2 = \left( m_1 + m_2 \right) \vec{v}_{12} \]
\[ (0.10 \text{ kg})(1000 \text{ m/s}) + (999.9 \text{ kg})(0 \text{ m/s}) = (1000 \text{ kg})\vec{v}_{12} \]
\[ \vec{v}_{12} = \frac{100 \text{ kg} \cdot \text{m/s}}{1000 \text{ kg}} \]
\[ = 0.10 \text{ m/s} \]

(b) In this case \( \vec{p}_{\text{total}} = \vec{p}_{\text{total}} \) also.
\[
\vec{p}_{\text{total}} = \vec{p}_{\text{total}} \\
\frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2} = \frac{m_1 \vec{v}'_1 + m_2 \vec{v}'_2}{m_1 + m_2}
\]

\[
(0.10 \text{ kg})(1000 \text{ m/s}) + 0 \text{ kg \cdot m/s} = (0.10 \text{ kg})(-1000 \text{ m/s}) + (1000 \text{ kg})\vec{v}'_2
\]

\[
\vec{v}'_2 = \frac{100 \text{ kg \cdot m/s} + 100 \text{ kg \cdot m/s}}{1000 \text{ kg}}
\]

\[
= 0.20 \text{ m/s}
\]

31. ANS:

\[
\vec{p}_{\text{total}} = \vec{p}_{\text{total}} \\
\frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2} = \frac{(m_1 + m_2) \vec{v}_{12}}{m_1 + m_2}
\]

\[
(2.0 \times 10^{-3} \text{ kg})(300 \text{ m/s}) + (1.5 \text{ kg})(0 \text{ m/s}) = \left(2.0 \times 10^{-3} \text{ kg} + 1.5 \text{ kg}\right)\vec{v}_{12}
\]

\[
\vec{v}_{12} = \frac{0.60 \text{ kg \cdot m/s}}{1.502 \text{ kg}}
\]

\[
\vec{v}_{12} = 0.40 \text{ m/s [forward]}
\]

32. ANS:

\[
W = F \Delta d
\]

\[
= (500 \text{ N})(4.0 \text{ m})
\]

\[
= 2000 \text{ J, or } 2.0 \times 10^3 \text{ J}
\]

33. ANS:

\[
W = \vec{F} \cdot \Delta \vec{d}
\]

\[
= (72 \text{ N})(0.40 \text{ m})(\cos 90^\circ)
\]

\[
= 0 \quad (\text{since } \cos 90^\circ = 0)
\]

The force changes the object’s direction but does not cause it to undergo a displacement in the direction of the force, and has no effect on its speed.

34. ANS:

(a)
Only the horizontal component of the applied force does work on the lawnmower.

(b)

\[ W = \vec{F} \cdot \Delta \vec{d} \]
\[ = F \Delta d \cos \theta \quad \text{(where } \theta = 45^\circ) \]
\[ = (141 \text{ N})(8.5 \text{ m})(0.707) \]
\[ = 847 \text{ J or } 8.5 \times 10^2 \text{ J} \]

No work is done since the force and displacement are at right angles—neither has a component in the direction of the other.

\[ \Delta d = 2 \pi R = 2 \pi (10 \text{ m}) \]

\[ W = \vec{F} \cdot \Delta \vec{d} \]
\[ = F \Delta d \cos \theta \]
\[ = (50 \text{ N})(20 \pi \text{ m})(\cos 90^\circ) \]
\[ = 0 \text{ J} \]

35. ANS:
From a force-displacement graph, the work done in any interval is the area under that interval on the \( F \cdot d \) graph.

<table>
<thead>
<tr>
<th>Interval</th>
<th>Area Under Curve</th>
<th>Total Work Done to End of Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 1 m</td>
<td>(3 N)(1 m) = 3 J</td>
<td>3 J</td>
</tr>
<tr>
<td>1 – 2 m</td>
<td>(2 N)(1 m) = 2 J</td>
<td>5 J</td>
</tr>
<tr>
<td>2 – 3 m</td>
<td>(0 N)(1 m) = 0 J</td>
<td>5 J</td>
</tr>
<tr>
<td>3 – 4 m</td>
<td>(–1 N)(1 m) = –1 J</td>
<td>4 J</td>
</tr>
</tbody>
</table>
Therefore, the graph of $W$ versus $d$ is:

![Graph of W versus d]

Note: curved shape of graph due to sloping portion of $F$-$d$ graph.

36. ANS:

$$E_k = \frac{1}{2} mv^2$$

$$= \frac{1}{2} \left( 12 \text{ kg} \right) \left( 2.0 \text{ m/s} \right)^2$$

$$= 24 \text{ J}$$

37. ANS:

$$E_k = \frac{1}{2} mv^2$$

$$v^2 = \frac{2E_k}{m}$$

$$= \frac{2 \left( 9.2 \times 10^{-18} \text{ J} \right)}{9.1 \times 10^{-31} \text{ kg}}$$

$$= 20.2 \times 10^{12} \text{ (m/s)}^2$$

$$v = 4.5 \times 10^6 \text{ m/s}$$
(a) \[ E_k = \frac{1}{2}mv^2 \]
\[ = \frac{1}{2} (4.0 \, \text{kg})(6.0 \, \text{m/s})^2 \]
\[ = 72 \, \text{J} \]

(b) The kinetic energy of the hammer changes from 72 J to zero, so that \(-72 \, \text{J}\) of work must have been done on it. This work was done by the post, pushing on the hammer with a constant force of \(F_{av} \uparrow\), as the hammer (and post) moved down 10 cm.

\[ W = \Delta E_k \]
\[ = E_{k2} - E_{k1} \]
\[ = 0 - 72 \, \text{J} \]
\[ = -72 \, \text{J} \]

But, \[ W = \vec{F} \cdot \Delta \vec{d} \]
\[ -72 \, \text{J} = F_{av} (0.10 \, \text{m})(\cos 180^\circ) \]
\[ = -F_{av} (0.10 \, \text{m}) \quad \text{(since \( F_{av} \) and \( \Delta \vec{d} \) are in opposite directions)} \]
\[ \vec{F}_{av} = 7.2 \times 10^2 \, \text{N} \uparrow \]

Therefore, the force exerted by the hammer on the post must also be \(7.2 \times 10^2 \, \text{N}\), but in the opposite direction (i.e., down—Newton’s Third Law). This ignores the gravitational force on the hammer, which would add another 39 N.

REF: K/U OBJ: 4.2 LOC: EM1.05 KEY: FOP 9.2, p.341
MSC: SP
39. ANS:
\[ E_k = \frac{p^2}{2m} \]
\[ = \frac{(30 \, \text{kg} \cdot \text{m/s})^2}{2(15 \, \text{kg})} \]
\[ = 30 \, \text{J} \]

MSC: SP
40. ANS:
\[ p = \sqrt{2mE_k} \]
\[ = \sqrt{2 \left( 9.1 \times 10^{-31} \text{ kg} \right) \left( 5.0 \times 10^3 \text{ eV} \right) \left( 1.6 \times 10^{-19} \text{ J/eV} \right)} \]
\[ = 3.8 \times 10^{-23} \text{ kg} \cdot \text{m/s} \]

ANS:

(a) \[ E_k = \frac{1}{2} m\nu^2 \]
\[ = \frac{1}{2} \left( 800 \text{ kg} \right) \left( 15 \text{ m/s} \right)^2 \]
\[ = 9.0 \times 10^4 \text{ J} \]

(b) \[ E_k = \frac{1}{2} m\nu^2 \]
\[ = \frac{1}{2} \left( 2.0 \times 10^{-3} \text{ kg} \right) \left( 500 \text{ m/s} \right)^2 \]
\[ = 2.5 \times 10^2 \text{ J} \]

(c) \[ E_k = \vec{F} \cdot \Delta \vec{d} \]
\[ = (8.0 \text{ N})(30 \text{ m}) \]
\[ = 24 \text{ J} \]

(d) \[ \nu = \frac{2\pi r}{T} \]
\[ = \frac{2\pi(2.0 \text{ m})}{\frac{\pi}{4}} \]
\[ = 16 \text{ m/s} \]

\[ \therefore E_k = \frac{1}{2} m\nu^2 \]
\[ = \frac{1}{2} \left( 0.25 \text{ kg} \right) \left( 16 \text{ m/s} \right)^2 \]
\[ = 32 \text{ J} \]
42. ANS:

(a) \( E_k = \frac{1}{2} m v^2 \)
\[ = \frac{1}{2} \left( 0.250 \text{ kg} \right) \left( 40 \text{ m/s} \right)^2 \]
\[ = 2.0 \times 10^2 \text{ J} \]

(b) \( W = \Delta E_k \)
\[ = E_{k_2} - E_{k_1} \]
\[ = 0 \text{ J} - 2.0 \times 10^2 \text{ J} \]
\[ = -2.0 \times 10^2 \text{ J} \]

(c) \( W = \vec{F} \cdot \Delta \vec{d} \)
\[ \therefore F_{\text{avg}} = \frac{W}{\Delta d} \]
\[ = \frac{-2.0 \times 10^2 \text{ J}}{0.25 \text{ m}} \]
\[ = -8.0 \times 10^2 \text{ N} \]

43. ANS:

(a) \( E_k(\text{total}) = E_k(\text{cart 1}) + E_k(\text{cart 2}) \)
\[ = \frac{1}{2} m v_1^2 + 0 \text{ J} \]
\[ = \frac{1}{2} \left( 5.0 \text{ kg} \right) \left( 2.0 \text{ m/s} \right)^2 \]
\[ = 10 \text{ J} \]
(b) $E_k(\text{at } x = 0.05 \text{ m}) = E_k(\text{initial}) - \left( \text{area under } F-x \text{ graph from } 0.20 \text{ m to } 0.05 \text{ m} \right)$

$$= 10 \text{ J} - \left[ \frac{1}{2} (15 \text{ N} + 30 \text{ N})(0.10 \text{ m}) + (0.05 \text{ m})(30 \text{ N}) \right]$$

$$= 10 \text{ J} - [2.25 \text{ J} + 1.5 \text{ J}]$$

$$= 6.25 \text{ J}$$

$$\approx 6.3 \text{ J}$$

(c) The 3.7 J of kinetic energy that has disappeared is stored in the collision mechanism as potential energy, to be released later as the collision proceeds.

REF: K/U, C OBJ: 5.3 LOC: EM1.03 KEY: FOP 9.5, p.351
MSC: SP

44. ANS:

(a) $W = \text{area under } F-d \text{ graph from } 0 \text{ to } 2.0 \text{ m}$

$$= \frac{1}{2} (2.0 \text{ m})(10 \text{ N})$$

$$= 10 \text{ J}$$

(b) $W = \text{area under } F-d \text{ graph from } 0 \text{ to } 3.0 \text{ m}$

$$= 10 \text{ J} + (1 \text{ m})(10 \text{ N})$$

$$= 20 \text{ J}$$

(c) $E_k = \frac{1}{2} m \nu^2$

$$\nu = \sqrt{\frac{2E_k}{m}}$$

$$= \sqrt{\frac{(2)(20 \text{ J})}{10 \text{ kg}}}$$

$$= 2.0 \text{ m/s}$$

REF: K/U OBJ: 4.2 LOC: EM1.05 KEY: FOP 9.9, p.368
MSC: P

45. ANS:

$$p = \sqrt{2mE_k}$$

$$= \sqrt{(2)(5.0 \text{ kg})(5.0 \times 10^2 \text{ J})}$$

$$= 70.7 \text{ kg} \cdot \text{m/s}, \text{ or } 71 \text{ kg} \cdot \text{m/s}$$
46. ANS:

\[ W = \Delta E_x \]

\[ F \Delta d = E'_x - E_x \]

\[ = 0 J - \frac{1}{2}mv^2 \]

\[ F(1.5 \text{ m}) = -\frac{1}{2} (1000 \text{ kg})(20 \text{ m/s})^2 \]

\[ F = -1.3 \times 10^5 \text{ N} \]

\[ \frac{F}{F_g} = \frac{1.3 \times 10^5 \text{ N}}{(1000 \text{ kg})(9.8 \text{ N/kg})} \]

\[ \approx 13.3 \]

47. ANS:

\[ E_x = \frac{1}{2}mv^2 \]

\[ = \frac{1}{2} \left( 9.1 \times 10^{-31} \text{ kg} \right) \left( 1.0 \times 10^7 \text{ m/s} \right)^2 \]

\[ = 4.6 \times 10^{-17} \text{ J} \]

48. ANS:

(a) \[ W = \vec{F} \cdot \Delta \vec{d} \]

\[ = (30 \text{ N})(4.0 \text{ m}) \]

\[ = 1.2 \times 10^2 \text{ J} \]

(b) \[ W = \vec{F} \cdot \Delta \vec{d} \]

\[ = (1500 \text{ N})(12 \text{ m}) \]

\[ = 1.8 \times 10^4 \text{ J} \]
(c) $W = \vec{F} \cdot \Delta \vec{d}$
\[ - (150 \text{ N})(8.0 \text{ m}) \]
\[ = 1.2 \times 10^5 \text{ J} \]

(d) $F = mg$
\[ = (1.6 \text{ kg})(9.8 \text{ m/s}^2) \]
\[ = 1.57 \times 10^1 \text{ N} \]

$W = \vec{F} \cdot \Delta \vec{d}$
\[ = \left(1.57 \times 10^1 \text{ N}\right)(20 \text{ m}) \]
\[ = 3.1 \times 10^2 \text{ J} \]

(e) $W = \vec{F} \cdot \Delta \vec{d}$
\[ - (120 \text{ N})(2.5 \text{ m}) \]
\[ = 3.0 \times 10^2 \text{ J} \]

(It is the applied force, not the net force, that is used to calculate the work done by the appropriate agent; in this case, the girl.)

REF: K/U OBJ: 4.1 LOC: EMV.01 KEY: FOP 9.1, p.334
MSC: P
ANS:

50. ANS:
Height of fall required for gravitational force to do 240 J of work on the hammer is $\Delta h$, where
\[\Delta h = \frac{W}{F}\]
\[= \frac{W}{mg}\]
\[= \frac{(240 \text{ J})}{(10 \text{ kg})(9.8 \text{ N/kg})}\]
\[= 2.4 \text{ m}\]

REF: K/U OBJ: 4.4 LOC: EM1.05 KEY: FOP 9.1, p.334
MSC: P
ANS:
(a) \[ E_k = \frac{1}{2} mv^2 \]
\[ = \frac{1}{2} (60 \text{ kg})(12 \text{ m/s})^2 \]
\[ = 4.3 \times 10^3 \text{ J} \]

(b) Work done to stop sled = \( \Delta E_k \)
\[ F \Delta d = \Delta E_k = -4.3 \times 10^3 \text{ J} \]
\[ \Delta d = \frac{-4.3 \times 10^3 \text{ J}}{-540 \text{ N}} \]
\[ = 8.0 \text{ m} \]

(c) \[ W = \Delta E_k \]
\[ = E'_k - E_k \]
\[ = 0 \text{ J} - 4.3 \times 10^3 \text{ J} \]
\[ = -4.3 \times 10^3 \text{ J} \]

51. ANS:

(a) \( E_k \) (final) = total work done on trolley
\[ = F_1 \Delta d_1 + F_2 \Delta d_2 \](since \( \vec{F} \) and \( \Delta \vec{d} \) are in the same direction)
\[ = (50 \text{ N})(4.0 \text{ m}) + (20 \text{ N})(2.0 \text{ m}) \]
\[ = 200 \text{ J} + 40 \text{ J} \]
\[ = 240 \text{ J}, \text{ or } 2.4 \times 10^2 \text{ J} \]

(b) \[ v = \sqrt{\frac{2E_k}{m}} \]
\[ = \sqrt{\frac{2(2.4 \times 10^2 \text{ J})}{2.0 \text{ kg}}} \]
\[ = 1.5 \times 10^1 \text{ m/s}, \text{ or } 15 \text{ m/s} \]
52. **ANS:**

\[ W = F \Delta d \]

where \[ \Delta d = \left( \frac{v_1 + v_2}{2} \right) \Delta t \]

\[ = \left( \frac{0 + 8.0 \text{ m/s}}{2} \right)(2.5 \text{ s}) \]

\[ = 10 \text{ m} \]

Therefore, \[ W = (20 \text{ N})(10 \text{ m}) \]

\[ = 200 \text{ J}, \text{ or } 2.0 \times 10^2 \text{ J} \]

**REF:** K/U **OBJ:** 4.1 **LOC:** EMV.01 **KEY:** FOP 9.1, p.332

53. **ANS:**

If the spring obeys Hooke’s Law,

\[ F = kx \]

\[ x = \frac{F}{k} \]

But, \[ F = mg \]

\[ = (0.25 \text{ kg})(9.8 \text{ m/s}^2) \]

\[ = 2.4 \text{ N} \]

Therefore, \[ x = \frac{2.4 \text{ N}}{48 \text{ N/m}} \]

\[ = 0.050 \text{ m}, \text{ or } 5.0 \text{ cm} \]

**REF:** K/U **OBJ:** 4.5 **LOC:** EM1.08 **KEY:** FOP 10.1, p.374

54. **ANS:**

\[ F = kx \]

\[ = (120 \text{ N/m})(0.30 \text{ m}) \]

\[ = 36 \text{ N} \]

**REF:** K/U **OBJ:** 4.5 **LOC:** EM1.08 **KEY:** FOP 10.1, p.374

55. **ANS:**
\[ F = kx \]
\[ = (600 \text{ N/m})(0.075 \text{ m}) \]
\[ = 45 \text{ N} \]

\[ m = \frac{F}{g} \]
\[ = \frac{45 \text{ N}}{9.8 \text{ N/kg}} \]
\[ = 4.6 \text{ kg} \]

REF: K/U OBJ: 4.5 LOC: EM1.08 KEY: FOP 10.1, p.374
MSC: P

56. ANS:
\[ F = kx \]
\[ k = \frac{F}{x} \]
\[ = \frac{mg}{x} \]
\[ = \frac{(40 \text{ kg})(9.8 \text{ N/kg})}{(0.12 \text{ m})} \]
\[ = 3.3 \times 10^3 \text{ N/m} \]

REF: K/U OBJ: 4.5 LOC: EM1.08 KEY: FOP 10.1, p.374
MSC: P

57. ANS:
\[ F = kx \]
\[ = (40 \text{ N/m})(0.080 \text{ m}) \]
\[ = 3.2 \text{ N} \]

Then, \[ a = \frac{F}{m} \]
\[ = \frac{3.2 \text{ N}}{2.0 \text{ kg}} \]
\[ = 1.6 \text{ m/s}^2 \]

REF: K/U OBJ: 4.5 LOC: EM1.08 KEY: FOP 10.1, p.374
MSC: P

58. ANS:
\[ E_s = \frac{1}{2} k x^2 \]
\[ = \frac{1}{2} (160 \, \text{N/m})(-0.080 \, \text{m})^2 \]
\[ = 0.51 \, \text{J} \]

The block loses kinetic energy by doing work that compresses the spring, and stores elastic potential energy in it.

\[- \Delta E_k(\text{block}) = \Delta E_s(\text{spring})\]

\[ \frac{1}{2} m v^2 = \frac{1}{2} k x^2 \]
\[ x^2 = \frac{m v^2}{k} \]
\[ = \frac{(2.5 \, \text{kg})(3.0 \, \text{m/s})^2}{360 \, \text{N/m}} \]
\[ = 0.0625 \, \text{m}^2 \]
\[ x = \pm 0.25 \, \text{m} \]

Since the spring is compressed, the + solution is inadmissible.

\[ E = \frac{1}{2} k x^2 \]
\[ k = \frac{2E}{x^2} \]
\[ = \frac{2(0.72 \, \text{J})}{(0.15 \, \text{m})^2} \]
\[ = 64 \, \text{N/m} \]

The block loses kinetic energy by doing work that compresses the spring, and stores elastic potential energy in it.

\[- \Delta E_k(\text{block}) = \Delta E_s(\text{spring})\]

\[ \frac{1}{2} m v^2 = \frac{1}{2} k x^2 \]
\[ x^2 = \frac{m v^2}{k} \]
\[ = \frac{(2.5 \, \text{kg})(3.0 \, \text{m/s})^2}{360 \, \text{N/m}} \]
\[ = 0.0625 \, \text{m}^2 \]
\[ x = \pm 0.25 \, \text{m} \]

Since the spring is compressed, the + solution is inadmissible.

\[ E = \frac{1}{2} k x^2 \]
\[ k = \frac{2E}{x^2} \]
\[ = \frac{2(0.72 \, \text{J})}{(0.15 \, \text{m})^2} \]
\[ = 64 \, \text{N/m} \]
\[ E = \frac{1}{2} k x^2 \]
\[ = \frac{1}{2} (80 \text{ N/m})(0.20 \text{ m})^2 \]
\[ = 1.6 \text{ J} \]

62. ANS:
\[ E = \frac{1}{2} k x^2 \]
\[ x = \sqrt{\frac{2E}{k}} \]
\[ = \sqrt{\frac{2(3.75 \text{ J})}{120 \text{ N/m}}} \]
\[ = 0.25 \text{ m} \]

63. ANS:
\[ |\Delta E_x| = |\Delta E_k| \]
\[ \frac{1}{2} k x^2 = \frac{1}{2} m v^2 \]
\[ v = \sqrt{\frac{k x^2}{m}} \]
\[ = \sqrt{\frac{(50 \text{ N/m})(0.20 \text{ m})^2}{5.0 \times 10^{-3} \text{ kg}}} \]
\[ = 20 \text{ m/s} \]

64. ANS:
(a) Work done = area under \( F-x \) graph from \( x = 0 \) to \( x = 0.16 \text{ m} \)
\[ = \frac{1}{2} (0.04 \text{ m})(4 \text{ N}) + (0.04 \text{ m})(4 \text{ N}) + (0.08 \text{ m}) \left( \frac{4 \text{ N} + 8 \text{ N}}{2} \right) \]
\[ = 0.08 \text{ J} + 0.16 \text{ J} + 0.48 \text{ J} \]
\[ = 0.72 \text{ J} \]
(b) \( E_{E}(\text{stored}) = 0.72 \text{ J} \)

(c) \( \frac{1}{2} m v^2 = 0.72 \text{ J} \)

\[
v = \sqrt{\frac{(2)(0.72 \text{ J})}{1.0 \text{ kg}}}
\]

\[= 1.2 \text{ m/s} \]

REF: K/U OBJ: 4.5 LOC: EM1.05 KEY: FOP 10.4, p.377
MSC: P

65. ANS:
(a) In orbit:
\[
\Delta E_g = \frac{GM_e m_s (r_2 - r_1)}{r_2 r_1}
\]

\[
= \frac{\left(6.67 \times 10^{-11} \text{ N \cdot m}^2/\text{kg}^2\right) \left(5.98 \times 10^{24} \text{ kg}\right) \left(1.99 \times 10^{30} \text{ kg}\right) \left(0.05 \times 10^{11} \text{ m}\right)}{\left(1.52 \times 10^{11} \text{ m}\right) \left(1.47 \times 10^{11} \text{ m}\right)}
\]

\[= 1.8 \times 10^{32} \text{ J} \]

(b) \( v \) is a maximum when \( E_k \) is a maximum, which occurs when \( E_g \) is a minimum. \( E_g \) is a minimum at perihelion. Then,

\[
\Delta E_k = -\Delta E_g = 1.8 \times 10^{32} \text{ J}
\]

MSC: P

66. ANS:
(a) \( E_g = -\frac{GM_e m_m}{r} \)

\[
= -\left(6.67 \times 10^{-11} \text{ N \cdot m}^2/\text{kg}^2\right) \left(5.98 \times 10^{24} \text{ kg}\right) \left(7.35 \times 10^{22} \text{ kg}\right)
\]

\[\div 3.85 \times 10^8 \text{ m} \]

\[= -7.61 \times 10^{28} \text{ J} \]

(b) For orbit: \( E_k = \frac{1}{2} |E_g| = 3.81 \times 10^{28} \text{ J} \)
\[ v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{(2)(3.81 \times 10^{26} \text{ J})}{7.35 \times 10^{-22} \text{ kg}}} = 1.02 \times 10^3 \text{ m/s} \]

(c) Total energy in orbit \( = \frac{1}{2} E_e \)
\[ = -3.81 \times 10^{26} \text{ J} \]

Therefore, the binding energy is \(3.81 \times 10^{26} \text{ J} \).

**ANS:**

(a) \( x = \frac{F}{k} \)
\[ = \frac{mg}{k} = \frac{(2.5 \text{ kg})(9.8 \text{ N/kg})}{200 \text{ N/m}} = 0.123 \text{ m, or 0.12 m} \]

(b) \( E_e = \frac{1}{2} kx^2 \)
\[ = \frac{1}{2} (200 \text{ N/m})(0.12 \text{ m})^2 = 1.5 \text{ J} \]
\[ k = \frac{F}{x} \]
\[ = \frac{5.0 \text{ N}}{0.10 \text{ m}} \]
\[ = 50 \text{ N/m} \]

For the collision:
\[ \frac{1}{2} m v^2 = \frac{1}{2} k x^2 \]
\[ x = \sqrt{\frac{m v^2}{k}} \]
\[ = \sqrt{\frac{(4.5 \text{ kg})(2.0 \text{ m/s})^2}{50 \text{ N/m}}} \]
\[ = 0.60 \text{ m, or } 60 \text{ cm} \]

**ANS:**
(a) \[ E_g = mg \Delta h \]
\[ = (30 \text{ kg})(9.8 \text{ N/kg})(4.0 \text{ m}) \]
\[ = 1176 \text{ J, or } 1.2 \times 10^3 \text{ J} \]

(b) \[ E_k = \frac{1}{2} m v^2 \]
\[ = \frac{1}{2} (30 \text{ kg})(2.5 \text{ m/s})^2 \]
\[ = 93.75 \text{ J, or } 94 \text{ J} \]

(c) \[ \Delta E = 94 \text{ J} - 1176 \text{ J} \]
\[ = -1082 \text{ J, or } -1.1 \times 10^3 \text{ J} \]
(d) \( \Delta E = W = \overrightarrow{F} \cdot \Delta \overrightarrow{d} \)

\[
\begin{align*}
\overrightarrow{F} &= \frac{\Delta E}{\Delta \overrightarrow{d}} \\
&= \frac{-1082 \text{ J}}{10.0 \text{ m}} \\
&= -1.1 \times 10^2 \text{ N}
\end{align*}
\]

REF: K/U OBJ: 4.4 LOC: EM1.05 KEY: FOP 10.4, p.387
MSC: P

70. ANS:
\( \Delta E_g = mg \Delta h \)
where \( \Delta h = 15(\pm 0.20 \text{ m}) \)
\[= 3.0 \text{ m} \]

Therefore, \( \Delta E_g = (50 \text{ kg})(9.8 \text{ N/kg})(3.0 \text{ m}) \)
\[= 1470 \text{ J}, \text{ or } 1.5 \times 10^3 \text{ J} \]

REF: K/U OBJ: 4.3 LOC: EMV.01 KEY: FOP 10.4, p.382
MSC: SP

71. ANS:
(a) \( E_{e_1} = \frac{1}{2} kx_1^2 \)
\[= \frac{1}{2} (50 \text{ N/m})(0.20 \text{ m})^2 \]
\[= 1.0 \text{ J} \]

(b) \( E_{e_2} = \frac{1}{2} kx_2^2 \)
\[= \frac{1}{2} (50 \text{ N/m})(0.60 \text{ m})^2 \]
\[= 9.0 \text{ J} \]

\[\therefore \Delta E = E_{e_2} - E_{e_1} \]
\[= 8.0 \text{ J} \]

(c) \( \Delta E_k = -\Delta E_e = 9.0 \text{ J} \)
\[ v = \sqrt{\frac{2E_k}{m}} \]
\[ = \sqrt{\frac{18.0 \text{ J}}{0.40 \text{ kg}}} \]
\[ = 6.7 \text{ m/s} \]

---

72. **ANS:**
\[ E_T = E_k + E_g \]
\[ = 5.0 \times 10^9 \text{ J} - 6.4 \times 10^9 \text{ J} \]
\[ = -1.4 \times 10^9 \text{ J} \]

Therefore, the binding energy is \( 1.4 \times 10^9 \text{ J} \).

---

73. **ANS:**
Consider \( A \) as 2 half-springs joined together, each stretching by 2 cm under the force \( mg \). Then, for each,
\[ k = \frac{F}{x} \]
\[ = \frac{mg}{2} \]
\[ = 1 \text{ cm} \]

In B, the force on each spring is \( \frac{mg}{2} \), therefore,
\[ x = \frac{F}{k} \]
\[ = \left( \frac{mg}{2} \right) \]
\[ = \left( \frac{mg}{2} \right) \]
\[ = 1 \text{ cm} \]
PROBLEM

74. ANS:
\[ \text{impulse} = \vec{F} \Delta t \]
\[ = (-1.4 \text{ N})(1.5 \text{ s}) \]
\[ = -2.1 \text{ N} \cdot \text{s} \]

But, impulse = \( \Delta \vec{p} \)
\[ = \vec{p}_2 - \vec{p}_1 \]
\[ = m \vec{v}_2 - m \vec{v}_1 \]
\[ \therefore \vec{v}_1 = \frac{\Delta \vec{p} + m \vec{v}_1}{m} \]
\[ = \frac{-2.1 \text{ kg} \cdot \text{m/s} + (0.50 \text{ kg})(2.4 \text{ m/s})}{(0.50 \text{ kg})} \]
\[ = -1.8 \text{ m/s}, \text{ which represents a final velocity of } 1.8 \text{ m/s [backward]} \]

REF: K/U OBJ: 5.1 LOC: EM1.01 KEY: FOP 8.2, p.297
MSC: P

75. ANS:
\[ \text{impulse} = \vec{F} \Delta t \]
\[ = (6.0 \text{ N})(0.50 \text{ s}) \]
\[ = 3.0 \text{ N} \cdot \text{s} \]

But, impulse = \( \Delta \vec{p} \)
\[ = \vec{p}_2 - \vec{p}_1 \]
\[ = m \vec{v}_2 - m \vec{v}_1 \]
\[ \therefore \vec{v}_1 = \frac{m \vec{v}_2 - \Delta \vec{p}}{m} \]
\[ = \frac{(2.0 \text{ kg})(4.5 \text{ m/s}) - (3.0 \text{ kg} \cdot \text{m/s})}{(2.0 \text{ kg})} \]
\[ = 3.0 \text{ m/s}, \text{ which represents an initial velocity of } 3.0 \text{ m/s [forward]} \]

REF: K/U OBJ: 5.1 LOC: EM1.01 KEY: FOP 8.2, p.297
MSC: P

76. ANS:
Since momentum is a vector quantity, directions are significant. We will assume that velocities to the right are positive, and to the left negative. Primed symbols indicate the value of a quantity after collision.
Before the collision:

\[ \vec{p}_1 = m_1 \vec{v}_1 \]

\[ = (6000 \text{ kg})(2.0 \text{ m/s}) \]

\[ = 1.2 \times 10^4 \text{ kg} \cdot \text{m/s} \text{ [right]} \]

\[ \vec{p}_2 = m_2 \vec{v}_2 \]

\[ = (3000 \text{ kg})(-3.0 \text{ m/s}) \]

\[ = -9.0 \times 10^3 \text{ kg} \cdot \text{m/s} \text{ [right]} \]

Therefore, \( \vec{p}_{total} = \vec{p}_1 + \vec{p}_2 \)

\[ = 1.2 \times 10^4 \text{ kg} \cdot \text{m/s} - 9.0 \times 10^3 \text{ kg} \cdot \text{m/s} \]

\[ = 3.0 \times 10^3 \text{ kg} \cdot \text{m/s} \text{ [right]} \]

After the collision:

\[ \vec{p}'_{total} = (m_1 + m_2) \vec{v}'_{12} \]

\[ = (9000 \text{ kg}) \vec{v}'_{12} \]

But, since the system is isolated, \( \Delta \vec{p} = 0 \).

Therefore,

\[ \vec{p}'_{total} = \vec{p}_{total} \]

\[ (9000 \text{ kg}) \vec{v}'_{12} = 3.0 \times 10^3 \text{ kg} \cdot \text{m/s} \]

\[ \vec{v}'_{12} = 0.33 \text{ m/s} \text{ [right]} \]

The pair of coupled cars moves off to the right (positive direction) at a speed of 0.33 m/s.

REF: K/U OBJ: 5.2 LOC: EM1.02 KEY: FOP 8.3, p.300
MSC: SP

ANS:

\[ \Delta \vec{p} = 0 \]

\[ \vec{p}'_{total} = \vec{p}_{total} \]

\[ \vec{p}'_b + \vec{p}'_r = \vec{p}'_b + \vec{p}'_r, \text{ where } r \text{ and } b \text{ represent the rifle and bullet respectively} \]

\[ m_b \vec{v}'_b + m_r \vec{v}'_r = m_b \vec{v}'_b + m_r \vec{v}'_r \]

If the bullet is assumed to be shot in the positive direction:
Since the two balls are considered to be an isolated system, we can equate their initial and final momenta. The final velocity of the 1.0 kg ball, $v'_1$, can be calculated as follows:

$$\overrightarrow{v}_1 = \frac{-15 \text{ kg} \cdot \text{m/s}}{5.0 \text{ kg}} = -3.0 \text{ m/s}$$

If we assume that vectors to the right are positive:

$$\overrightarrow{P}_{\text{total}} + \overrightarrow{P}'_{\text{total}} = \overrightarrow{P}_{\text{total}} + \overrightarrow{P}'_{\text{total}}$$

$$\overrightarrow{P}_1 + \overrightarrow{P}_2 = \overrightarrow{P}'_1 + \overrightarrow{P}'_2$$

The negative sign indicates that the 1.0 kg ball, initially moving to the right, rebounds and moves to the left at 0.40 m/s after the collision.

For the first team:

Let the velocity acquired by the cart with respect to Earth be $V$. The velocity of the three students relative to the Earth, $v$, will be $10 + V$.

$$\overrightarrow{P}'_{\text{total}} = \overrightarrow{P}'_{\text{total}} = 0$$

$$0 = m_c V + 3 m_s (10 + V)$$

$$= 120 V + 180(10 + V)$$

$$300 V = -1800$$

$$V = -6.0 \text{ m/s} \quad \text{and} \quad v = 4.0 \text{ m/s}$$

Note: The negative sign for $V$ indicates a velocity to the west.

For the second team:

Let the velocity of the cart after each successive student has jumped be $V_1$, $V_2$, and $V_3$, and let the velocities of the three students relative to the Earth be $v_1$, $v_2$, and $v_3$. After the first student has jumped:
Finally, after the third student has jumped:
\[ 0 = m_c V_3 + m_s V'_3 + m_s (10 + V_3) + m_c v_3 + m_s v_1 \]
\[ = 120 V_3 + 60 (10 + V_3) + 60 (5.5) + 60 (8.0) \]
\[ 180 V_3 = -1410 \]
\[ V_3 = -7.8 \text{ m/s, or } 7.8 \text{ m/s } [\text{W}] \text{ and } v_3 = 2.2 \text{ m/s} \]

As can be seen from the result, the team that chose to have its members jump separately was able to give the cart a greater velocity in the opposite direction.

For such a rocket,
\[ F_T = \nu_c \frac{\Delta m}{\Delta t} \]
\[ = \left(2.5 \times 10^4 \text{ m/s}\right) \left(10 \text{ kg/s}\right) \]
\[ = 2.5 \times 10^5 \text{ kg} \cdot \text{m/s}^2 \]
Then,
\[
\Delta t = \frac{1.4 \times 10^4 \text{ kg}}{2.5 \times 10^5 \text{ N}} (50 \text{ m/s}) = 2.8 \text{ s}
\]

81. ANS:
Let the directions of the bomb fragments be as indicated in the diagram.

Momentum is conserved in explosion:
\[
\vec{P}_{\text{total}} = \vec{P}_{\text{total}}' = \vec{P}_1' + \vec{P}_2' + \vec{P}_3' = 0
\]

Drawing a diagram, \( \vec{P}_3' = -\vec{P}_1' - \vec{P}_2' \)

Scale: 1 cm = 5 kg·m/s

\( \vec{P}_1' = 40 \text{ kg} \cdot \text{m/s [E]} \)

\( \vec{P}_2' = 136 \text{ kg} \cdot \text{m/s [E60°N]} \)

\( \vec{P}_3' = 13.2 \text{ cm} = 66 \text{ kg} \cdot \text{m/s [W28°S]} \) which is 148° counterclockwise from the 3.0 kg piece.

(b) If \( v_3' = 30 \text{ m/s}, \)
\[ m_3 = \frac{P_3}{v_2'} = \frac{66 \text{ kg} \cdot \text{m/s}}{30 \text{ m/s}} = 2.2 \text{ kg} \]

82. **ANS:**

In each case, momentum is conserved. The target (grey) ball \((m_2)\) is at rest, and \(m_1 \vec{v}_1 = m_1 \vec{v}_1' + m_2 \vec{v}_2'\)

But, \(m_1 = m_2\).

Therefore, \(\vec{v}_1 = \vec{v}_1' + \vec{v}_2'\), which can be measured from the photos in cm/flash.

(a) \(\vec{v}_1' = 2.0 \text{ cm/flash} [E]\)

\(\vec{v}_1' = 1.5 \text{ cm/flash} [E30^\circ S]\)

Then \(\vec{v}_2' = \vec{v}_1' - \vec{v}_1'\) and using a vector diagram:

Scale: 0.5 cm = 1.0 cm/flash

(b) \(\vec{v}_1' = 2.5 \text{ cm/flash} [E]\)

\(\vec{v}_2' = 1.0 \text{ cm/flash} [E60^\circ N]\)

\(\vec{v}_1' = \vec{v}_1 - \vec{v}_2'\)
(c) \( \vec{v}_1 = 1.75 \text{ cm/flash [E40°S]} \)
\[
\vec{v}_2 = 1.5 \text{ cm/flash [E50°N]}
\]
\[\vec{v}_1 = \vec{v}_1' + \vec{v}_2'\]
\[v_1 = 4.6 \text{ cm} = 2.3 \text{ cm/flash}\]

\[\vec{v}_1 = 2.3 \text{ cm/flash [131° from } \vec{v}_2'], \text{ or 2.3 cm/flash [140° from } \vec{v}_1']\]

---

83. **ANS:**

(a) impulse = \( F \Delta t \)

\[
= \left( 7.2 \times 10^3 \text{ N} \right) \left( 5.0 \times 10^{-4} \text{ s} \right)
= 3.6 \text{ N} \cdot \text{s}
\]

(b) \( m = 45 \text{ g} = 0.045 \text{ kg} \)

impulse = \( \Delta p \)

\[= p_2 - p_1, \text{ where } p_1 = 0\]

\[p_2 = \text{ impulse} \]
\[= m \nu_2\]

\[\therefore \nu_2 = \frac{\text{impulse}}{m}
= \frac{3.6 \text{ N} \cdot \text{s}}{0.045 \text{ kg}}
= 80 \text{ m/s}\]

---

84. **ANS:**

(a) \( p_{\text{shot}} = m \cdot \nu_\epsilon \)

\[= \left( 200 \text{ kg} \right) \left( 250 \text{ m/s} \right)
= 5.0 \times 10^4 \text{ kg} \cdot \text{m/s} \]
\[ p_{\text{cannon}} = -5.0 \times 10^4 \text{ kg} \cdot \text{m/s} \]

But impulse = \( \Delta p \)
\[ = p_2 - p_1 \]
\[ = -\left(-5.0 \times 10^4 \text{ N} \cdot \text{s}\right), \text{ since } p_2 = 0 \]

Finally, \( F = \frac{\text{impulse}}{\Delta t} \)
\[ = \frac{5.0 \times 10^4 \text{ N} \cdot \text{s}}{2.0 \text{ s}} \]
\[ = 2.5 \times 10^4 \text{ N} \]

(b) For the cannon,
\[ a = \frac{F}{m} \]
\[ = \frac{2.5 \times 10^4 \text{ N}}{2.0 \times 10^4 \text{ kg}} \]
\[ = 1.25 \text{ m/s}^2 \]

\[ \therefore \Delta \dot{x} = v_2 \Delta t - \frac{1}{2}a \Delta t^2 \]
\[ = 0 \text{ m} - \frac{1}{2} \left(1.25 \text{ m/s}^2\right)(2.0 \text{ s}) \]
\[ = -2.5 \text{ m} \]

**ANS:**
For the ball, assuming the initial direction of the ball is positive:
\[ p_1 = m_1 v_1 \]
\[ = (0.300 \text{ kg})(50 \text{ m/s}) \]
\[ = 15 \text{ kg} \cdot \text{m/s} \]
\[ p_2 = m_2 v_2 \]
\[ = 0.300 \text{ kg} \times (-100 \text{ m/s}) \]
\[ = -30 \text{ kg} \cdot \text{m/s} \]

\[ \therefore \text{ impulse } = \Delta p \]
\[ = p_2 - p_1 \]
\[ = -30 \text{ kg} \cdot \text{m/s} - 15 \text{ kg} \cdot \text{m/s} \]
\[ = -45 \text{ kg} \cdot \text{m/s} \]

\[ F_{av} = \frac{\text{impulse}}{\Delta t} \]
\[ = \frac{-0.45 \text{ kg} \cdot \text{m/s}}{0.020 \text{ s}} \]
\[ = -2.3 \times 10^3 \text{ N} \]

**ANS:**

(a) \[ p = mv \]
\[ = 60 \text{ kg} \times 20 \text{ m/s} \]
\[ = 1.2 \times 10^3 \text{ kg} \cdot \text{m/s} \]

(b) \[ F_{av} = \frac{\text{impulse}}{\Delta t} \]
\[ = \frac{\Delta p}{\Delta t} \]
\[ = \frac{p_2 - p_1}{\Delta t}, \text{ where } p_2 = 0 \]
\[ = \frac{-1.2 \times 10^3 \text{ kg} \cdot \text{m/s}}{30 \text{ s}} \]
\[ = -4.0 \times 10^2 \text{ N} \]
(c) \[ a = \frac{F}{m} \]
\[ = \frac{-4.0 \times 10^2 \text{ N}}{60 \text{ kg}} \]
\[ = -6.7 \text{ m/s}^2 \]

Then, \[ v_2^2 = v_1^2 + 2a\Delta d \quad \text{(where } v_2 = 0) \]

\[ \therefore \Delta d = -\frac{v_1^2}{2a} \]
\[ = -\frac{\left(20 \text{ m/s}^2\right)}{2\left(-6.7 \text{ m/s}^2\right)} \]
\[ = 30 \text{ m} \]

87. ANS:
(a) For uniformly accelerated motion,
\[ v_2^2 = v_1^2 + 2a\Delta d \]
\[ 0 = (400 \text{ m/s})^2 + 2a(0.10 \text{ m}) \]
\[ a = -\frac{(400 \text{ m/s})^2}{2(0.10 \text{ m})} \]
\[ = -8.0 \times 10^5 \text{ m/s}^2 \]

(b) Net force acting on bullet:
\[ F = ma \]
\[ = (0.050 \text{ kg})\left(-8.0 \times 10^5 \text{ m/s}^2\right) \]
\[ = -4.0 \times 10^4 \text{ N} \]
(c) impulse = change in momentum
\[ F \Delta t = m \Delta v \]
\[ \Delta t = \frac{m \Delta v}{F} \]
\[ = \frac{0.050 \text{ kg} \cdot (-400 \text{ m/s})}{-4.0 \times 10^4 \text{ N}} \]
\[ = 5.0 \times 10^{-4} \text{ s} \]

(d) impulse = \( F \Delta t \)
\[ = \left( -4.0 \times 10^4 \text{ N} \right) \left( 5.0 \times 10^{-4} \text{ s} \right) \]
\[ = -20 \text{ N} \cdot \text{s} \]

(e) \( p_1 = p_2 - \Delta p \)
\[ = p_2 - F \Delta t \]
\[ = 0 \text{ kg} \cdot \text{m/s} - (-20 \text{ N} \cdot \text{s}) \]
\[ = 20 \text{ kg} \cdot \text{m/s} \]

REF: K/U, 1  OBJ: 5.1  LOC: EM1.01  KEY: FOP 8.8, p.328
MSC: P

ANS:

(a) \( \alpha = \frac{\Delta v}{\Delta t} \)
\[ = \frac{100 \text{ m/s} - 400 \text{ m/s}}{4.0 \times 10^{-4} \text{ s}} \]
\[ = -7.5 \times 10^5 \text{ m/s}^2 \]

Then, \( F = ma \)
\[ = \left( 8.0 \times 10^{-3} \text{ kg} \right) \left( -7.5 \times 10^5 \text{ m/s}^2 \right) \]
\[ = -6.0 \times 10^3 \text{ N} \]
(b) $F \Delta d = \Delta E_k$

$$\Delta d = \frac{\Delta E_k}{F'}$$

$$= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

$$= \frac{1}{2} \left( 8.0 \times 10^{-3} \text{ kg} \right) \left[ (100 \text{ m/s})^2 - (400 \text{ m/s})^2 \right]$$

$$= -60 \times 10^3 \text{ N}$$

$$= 1.01 \times 10^{-1} \text{ m}, \text{ or } 10 \text{ cm}$$

ANS:

For the decelerating block and bullet:

$v_2 = 0$

$\Delta d = 0.60 \text{ m}$

$\Delta t = 2.0 \text{ s}$

$$\Delta d = v_2 \Delta t - \frac{1}{2} a \Delta t^2$$

$$= (0.60 \text{ m}) - 0 \text{ m} - \frac{1}{2} a (2.0 \text{ s})^2$$

$$a = -\frac{2(0.60 \text{ m})}{4.0 \text{ s}^2}$$

$$= -0.30 \text{ m/s}^2$$

$\therefore \: v_1 = v_2 - a \Delta t$

$$= 0 \text{ m/s} - \left( -0.30 \text{ m/s}^2 \right) (2.0 \text{ s})$$

$$= 0.60 \text{ m/s}$$

Momentum is conserved in the collision:
Momentum is conserved in the collision:
\[ m_1 \vec{v}_1 + m_2 \vec{v}_2 = (m_1 + m_2) \vec{v}_{12} \]

\[
(0.024 \text{ kg}) v_1 + (10 \text{ kg})(0 \text{ m/s}) = (10.024 \text{ kg})(0.60 \text{ m/s})
\]

\[
v_1 = \frac{(10.024 \text{ kg})(0.60 \text{ m/s})}{(0.024 \text{ kg})} = 251 \text{ m/s, or } 2.5 \times 10^2 \text{ m/s}
\]

\[ \vec{p}_{\text{total (before)}} = \vec{p}_1 + \vec{p}_2 \]

\[ -m_1 \vec{v}_1 + m_2 \vec{v}_2 \]

\[ = \left(2000 \text{ kg}\right)(24 \text{ m/s [E]}) + \left(3600 \text{ kg}\right)(10 \text{ m/s [S]}) \]

\[ = 4.8 \times 10^4 \text{ kg \cdot m/s [E]} + 3.6 \times 10^4 \text{ kg \cdot m/s [S]} \]

\[ |\vec{p}_{\text{total}}| = \sqrt{|\vec{p}_1|^2 + |\vec{p}_2|^2} \]

\[ = \sqrt{\left(4.8 \times 10^4 \text{ kg \cdot m/s}\right)^2 + \left(3.6 \times 10^4 \text{ kg \cdot m/s}\right)^2} \]

\[ = \sqrt{36 \times 10^8 \left(\text{kg \cdot m/s}\right)^2} \]

\[ = 6.0 \times 10^4 \text{ kg \cdot m/s} \]
\[ \theta = \tan^{-1} \left| \frac{\vec{P}_2}{\vec{P}_1} \right| \]

\[ = \tan^{-1} \frac{3.6 \times 10^4 \text{ kg} \cdot \text{m/s}}{4.8 \times 10^4 \text{ kg} \cdot \text{m/s}} \]

\[ = \tan^{-1} 0.75 \]

\[ = 37^\circ \]

\[ \vec{P}_{\text{total}} \text{(before)} = 6.0 \times 10^4 \text{ kg} \cdot \text{m/s} \]  

Since momentum is conserved,

\[ \vec{P}_{\text{total}} \text{(after)} = 6.0 \times 10^4 \text{ kg} \cdot \text{m/s} \]

\[ \therefore \vec{v}_{12} = \frac{\vec{P}_{\text{total}}}{m_1 + m_2} \]

\[ = \frac{6.0 \times 10^4 \text{ kg} \cdot \text{m/s}}{5.6 \times 10^3 \text{ kg}} \]

\[ = 1.07 \times 10^1 \text{ m/s}, \text{ or } 11 \text{ m/s} \]

ANS: 

Before the decay, \( \vec{P}_{\text{total}} = 0 \).

Since momentum is conserved in such an interaction, \( \vec{P}_{\text{total}} = 0 \) also.

\[ \vec{P}'_{\text{total}} = \vec{P}_1 + \vec{P}_2 + \vec{P}_3 \]

\[ = 9.0 \times 10^{-21} \text{ kg} \cdot \text{m/s [E]} + 4.8 \times 10^{-21} \text{ kg} \cdot \text{m/s [S]} + \vec{P}_3 \]

\[ = 0 \]

(a)
The direction taken by the residual nucleus is \([W28^\circ N]\).

\[
\begin{align*}
\theta &= \tan^{-1} \left( \frac{|\vec{p}_2|}{|\vec{p}_1|} \right) \\
&= \tan^{-1} \left( \frac{4.8 \times 10^{-21} \text{ km} \cdot \text{m/s}}{9.0 \times 10^{-21} \text{ km} \cdot \text{m/s}} \right) \\
&= \tan^{-1} 0.53 \\
&= 28^\circ
\end{align*}
\]

(b) \[|\vec{p}_3| = \sqrt{|\vec{p}_1|^2 + |\vec{p}_2|^2} \]

\[
= \sqrt{ \left( 9.0 \times 10^{-21} \text{ kg} \cdot \text{m/s} \right)^2 + \left( 4.8 \times 10^{-21} \text{ kg} \cdot \text{m/s} \right)^2 }
\]

\[
= \sqrt{ 104 \times 10^{-42} \text{ (kg} \cdot \text{m/s})^2 }
\]

\[
= 1.02 \times 10^{-20} \text{ kg} \cdot \text{m/s}
\]

\[
= 1.0 \times 10^{-20} \text{ kg} \cdot \text{m/s}
\]

(c) \[\vec{v}_3 = \frac{\vec{p}_3}{m_3}\]

\[
= \frac{1.02 \times 10^{-20} \text{ kg} \cdot \text{m/s} [W28^\circ N]}{3.6 \times 10^{-25} \text{ kg}}
\]

\[
= 2.8 \times 10^4 \text{ m/s} [W28^\circ N]
\]
92. ANS:
Assume the ball is moving to the right, and deflect at [30° up] as a result of the collision.

Before the collision:
\[ \vec{P}_{\text{total}} = \vec{P}_1 + \vec{P}_2 \]
\[ = m_1 \vec{v}_1 + m_2 \vec{v}_2 \]
\[ = (0.50 \text{ kg})(2.0 \text{ m/s})[\text{R}] + 0 \text{ kg} \cdot \text{m/s} \]
\[ = 1.0 \text{ kg} \cdot \text{m/s} \text{ [R]} \]

After the collision (since momentum is conserved):
\[ \vec{P}_{\text{total}} = \vec{P}_{\text{total}}' = 1.0 \text{ kg} \cdot \text{m/s} \text{ [R]} \]

But \[ \vec{P}_{\text{total}}' = m_1 \vec{v}_1' + m_2 \vec{v}_2' \]
\[ = 0.75 \text{ kg} \cdot \text{m/s}[\text{R}30^\circ \text{U}] + (0.30 \text{ kg}) \left( \frac{0.75 \text{ m/s}[\text{R}30^\circ D]}{0.30} \right) \]

Taking components in the L-R direction:
\[ 1.0 = 0.75 \cos 30^\circ + 0.30 \nu'_2 \cos \theta \quad (1) \]

Taking components in the U-D direction:
\[ 0 = 0.75 \sin 30^\circ - 0.30 \nu'_2 \sin \theta \quad (2) \]

From (1)
\[ \nu'_2 \cos \theta = \frac{1 - 0.75(0.866)}{0.30} \]
\[ = 1.168 \]

From (2)
\[ \nu'_2 \sin \theta = \frac{(0.75)(0.500)}{0.30} \]
\[ = 1.25 \]
\[
\tan \theta = \frac{v_2' \sin \theta}{v_2' \cos \theta}
\]
\[
= \frac{1.25}{1.168}
\]
\[
= 1.070
\]
\[
\therefore \ \theta = 47^\circ
\]

Then, \( v_2' = \frac{1.168}{\cos 47^\circ} \)
\[
= 1.7 \text{ m/s}
\]
\[
\vec{v}_2' = 1.7 \text{ m/s} \ [R47^\circ D]
\]

ANS:
Assume original direction of capsule is east.

Before the projectile is fired:
\[
\vec{P}_{\text{total}} = (m_1 + m_2) \vec{v}_1
\]
\[
= (3000 \text{ kg} + 25 \text{ kg}) (200 \text{ m/s} \ [E])
\]
\[
= 6.05 \times 10^5 \text{ kg} \cdot \text{m/s} \ [E]
\]

After the projectile is fired:
\[
\vec{P}'_{\text{total}} = \vec{P}_1' + \vec{P}_2'
\]
\[
= m_1 \vec{v}_1' + m_2 \vec{v}_2'
\]

Taking components in \( x \)-\( y \) directions:
\[
\vec{P}'_{\text{total}} = m_1 v_{1x}' + m_2 v_{2x}'
\]
\[
= (25 \text{ kg}) (0 \text{ m/s}) + (3000 \text{ kg}) (v_2' \cos \theta)
\]
\[ p_{\text{total}_y} = m_1 v'_{1y} + m_2 v'_{2y} \]
\[ = (25 \text{ kg})(2000 \text{ m/s}) - (3000 \text{ kg})(v'_2 \sin \theta) \]

Since \( \vec{P}_{\text{total}} = \vec{P}'_{\text{total}} \),

x components: \( 3000 v'_2 \cos \theta = 6.05 \times 10^5 \) \hspace{1cm} (1)

y components: \( 3000 v'_2 \sin \theta = 50000 \) \hspace{1cm} (2)

\[ \tan \theta = \frac{\sin \theta}{\cos \theta} \]
\[ = \frac{50000}{6.05 \times 10^5} \]
\[ = 0.0826 \]
\[ \theta = 4.76^\circ \]

Then, \( v'_2 = \frac{50000}{3000 \sin 4.76^\circ} \)

\[ = 2.02 \times 10^2 \text{ m/s} \]

REF: K/U  
OBJ: 5.4  
MSC: P  
94. ANS:  
For the boy: 

Before

\[ \begin{array}{c}
\begin{array}{c}
2M_2 \\

M_1
\end{array}
\end{array} \]

After

\[ \begin{array}{c}
\begin{array}{c}
(5 - V) \\

M_1
\end{array}
\end{array} \]

\[ \begin{array}{c}
\begin{array}{c}
2M_3 \\

V
\end{array}
\end{array} \]

Since momentum is conserved, \( \vec{P}_{\text{total}} = \vec{P}'_{\text{total}} \)
\[ m_1 \nu - 2m_2 (5 - \nu) = 0 \]
\[ 100 \nu + 20(\nu - 5) = 0 \]
\[ 120\nu - 100 \]
\[ \nu = 0.83 \text{ m/s} \]

For the girl:

Before

![Diagram showing the initial position of the system](image)

After 1

\[ (5 - \nu_1) \]
\[ M_2 \]

![Diagram showing the system after the first ball is thrown](image)

After 2

\[ (5 - \nu_2) \]
\[ M_2 \]

![Diagram showing the system after the second ball is thrown](image)

After the first ball is thrown:

\[ \vec{P}_{\text{total}} = \vec{P}'_{\text{total}} \]

\[ 0 = (m_1 + m_2) \nu_1 - m_2 (5 - \nu_1) \]

\[ = 110 \nu_1 + 10(\nu_1 - 5) \]

\[ \nu_1 = \frac{-50}{120} \]

\[ = 0.42 \text{ m/s} \]

After the second ball is thrown:

\[ \vec{P}_T = \vec{P}'_T \]
\[(m_1 + m_2)v_1 = m_1 v_2 - m_2 (5 - v_2)\]
\[(110)(0.42) = 100v_2 + 10(v_2 - 5)\]
\[100v_2 = 46 + 50\]
\[v_2 = 0.87 \text{ m/s}\]

The arrow is launched with a horizontal and vertical components of velocity given by:
\[v_h = v_0 \cos 30^\circ\]
\[= (40 \text{ m/s})(0.8660)\]
\[= 34.6 \text{ m/s}\]

\[v_v = v_0 \sin 30^\circ\]
\[= (40 \text{ m/s})(0.500)\]
\[= 20 \text{ m/s}\]

For the time before impact:
\[v_2 = v_1 + \alpha \Delta t\]

\[\Delta t = \frac{v_2 - v_1}{\alpha}\]
\[= \frac{0 \text{ m/s} - 20 \text{ m/s}}{-9.8 \text{ m/s}^2}\]
\[= 2.04 \text{ s}\]

At the moment of impact, the arrow has a horizontal velocity:
\[v_h = 34.6 \text{ m/s} \text{ and momentum is conserved}\]
\[ \tilde{P}_{\text{total}} = \tilde{P}'_{\text{total}} \]
\[ m_a \tilde{v}_a = \left( m_a + m_b \right) \tilde{v}_b \]
\[ \left( 0.500 \text{ kg} \right) \left( 34.6 \text{ m/s} \right) = \left( 0.500 \text{ kg} + 2.0 \text{ kg} \right) \nu_{ab} \]
\[ \nu_b = 6.9 \text{ m/s} \]

Since the time taken to fall to the ground is the same as the time taken for the arrow to rise,
\[ \Delta d_h = \nu_h \Delta t \]
\[ = \left( 6.9 \text{ m/s} \right) \left( 2.04 \text{ s} \right) \]
\[ = 14 \text{ m} \]

REF: K/U OBJ: 5.2 LOC: EM1.03 KEY: FOP 8.8, p.331
MSC: P

Assume the dog starts at the centre of the raft. (This is not necessary, but simplifies the algebra.) The centre of mass of the system is then 20 m from shore, and its momentum is zero. Therefore, the centre of mass of the system remains at rest 20 m from shore. If the centre of the raft moves \( y \) away from shore, then, taking torques about the balance point 20 m from shore:
\[ m_{\text{ref}}(y) = m_{\text{dog}} \left( 8.00 - y \right) \]
\[ \left( 40.0 \text{ kg} \right)y = \left( 10.0 \text{ kg} \right) \left( 8.00 - y \right) \]
\[ 50.0y = 80.0 \]
\[ y = 1.6 \text{ m} \]

Therefore after the walk, the distance of the dog from the shore is
\[ 20.0 \text{ m} + 1.6 \text{ m} - 8.0 \text{ m} = 13.6 \text{ m} \]

REF: K/U KEY: FOP 8.8, p.331 MSC: P

ANS:
After the first jump:

\[
\overrightarrow{P}_{\text{total}} = \overrightarrow{P}'_{\text{total}} = 0
\]

\[
m_1 (5.0 - \nu_1) + (m_2 + m_3)(-\nu_1) = 0
\]

\[
(100 \text{ kg})(5.0 \text{ m/s} - \nu_1) - (400 \text{ kg})(\nu_1) = 0
\]

\[
500 \text{ kg} \cdot \text{m/s} = (500 \text{ kg})(\nu_1)
\]

\[
\nu_1 = 1.0 \text{ m/s} \ [S]
\]

After the second jump:

\[
\overrightarrow{P}_{\text{total}} = \overrightarrow{P}'_{\text{total}} = 0
\]

\[
m_1 (5.0 - \nu_1) - m_2 (5.0 - \nu_2) + m_3 \nu_2 = 0
\]

\[
400 \text{ kg} \cdot \text{m/s} - (100 \text{ kg})(5.0 \text{ m/s} - \nu_2) + (300 \text{ kg})(\nu_2) = 0
\]

\[
(400 \text{ kg})(\nu_2) = 100 \text{ kg} \cdot \text{m/s}
\]

\[
\nu_2 = 0.25 \text{ m/s} \ [N]
\]
\[ \vec{P}_{\text{total}} = \vec{P}_{\text{total}} \]

\[ m_1 \vec{v}_1 + m_2 \vec{v}_2 = (m_1 + m_2) \vec{v}_{12} \]

\[ (1000 \text{ kg})(50 \text{ m/s}) + (2000 \text{ kg})(0 \text{ m/s}) - (3000 \text{ kg}) \vec{v}_{12} \]

\[ \vec{v}_{12} = \frac{50000 \text{ kg} \cdot \text{m/s}}{3000 \text{ kg}} \]

\[ = \frac{50}{3} \text{ m/s} \]

Then, the distance required by plane to slow down from 50 m/s to \( \frac{50}{3} \) m/s is given by

\[ v_2^2 = v_1^2 + 2a \Delta d \]

\[ a = \frac{F^\prime}{m} \]

\[ = \frac{1}{4} mg \]

\[ = \frac{m}{4} \]

\[ = \frac{-g}{4} \]

\[ = -2.45 \text{ m/s}^2 \]

\[ \therefore \Delta d = \frac{v_2^2 - v_1^2}{2a} \]

\[ = \frac{\left( \frac{50}{3} \text{ m/s} \right)^2 - (50 \text{ m/s})^2}{2 \left(-2.45 \text{ m/s}^2 \right)} \]

\[ = 453 \text{ m} \]

But, this is the distance relative to the water. The boat also moves forward as the plane slows down. For its distance,

\[ v_2^2 = v_1^2 + 2a \Delta d \]
\[ a = \frac{\vec{F}}{m_2} \]
\[ = \frac{1}{4} \left( \frac{m_1 g}{m_2} \right) \]
\[ = \frac{1}{4} \left( \frac{1000 \text{ kg}}{2000 \text{ kg}} \right) (9.8 \text{ N/kg}) \]
\[ = 1.225 \text{ m/s}^2 \]

Then, \[ \Delta d = \frac{v_2^2 - v_1^2}{2a} \]
\[ = \frac{(50 / 3 \text{ m/s})^2 - 0 \text{ m/s}}{2 \left( 1.225 \text{ m/s}^2 \right)} \]
\[ = 113 \text{ m} \]

The minimum length of barge is 453 m – 113 m = 340 m or 3.4 \times 10^2 m.

REF: K/U OBJ: 5.1 LOC: EM1.01 KEY: FOP 8.2, p.296
MSC: SP
99. ANS:
\[ \vec{F} \Delta t = m \Delta \vec{v} \]
\[ = m \vec{v}_2 - m \vec{v}_1 \]
\[ = m \vec{v}_2 \]

Therefore \[ \vec{v}_2 = \frac{\vec{F} \Delta t}{m} \]
\[ = \frac{(75 \text{ N})(2.0 \text{ s})}{80 \text{ kg}} \]
\[ = 1.9 \text{ m/s} \text{ [in the opposite direction of } \vec{F} \text{]} \]

REF: K/U OBJ: 5.1 LOC: EM1.01 KEY: FOP 8.2, p.296
MSC: SP
100. ANS:
(a) Momentum is conserved when he throws the ball, \[ \vec{p}_\text{total} = \vec{p}'_\text{total} = 0. \]
\[ 0 = m_1 \vec{v}_1' + m_2 \vec{v}_2' \]
\[ 0 = (0.200 \text{ kg})(25 \text{ m/s}) + (80 \text{ kg})(\vec{v}_2') \]
\[ \vec{v}_2' = \frac{- (0.200 \text{ kg})(25 \text{ m/s})}{(80 \text{ kg})} \]
\[ = -6.25 \times 10^{-2} \text{ m/s} \]
\[ \vec{v}_2' = 6.3 \times 10^{-2} \text{ m/s [backward]} \]

(b) \[ \vec{F} \Delta t = \Delta \vec{p} \]
\[ \vec{F} = \frac{\Delta p}{\Delta t} \]
\[ = \frac{m \Delta \vec{v}}{\Delta t} \]
\[ = (6)(80 \text{ kg})(-6.25 \times 10^{-2} \text{ m/s}) \]
\[ = 5.0 \text{ s} \]
\[ = -6.0 \text{ N} \]
\[ \vec{F} = 6.0 \text{ N [backward]} \]

REF: K/U OBJ: 5.2 LOC: EM1.02 KEY: FOP 8.8, p.329

MSC: P

101. ANS:
\[ a = \frac{\vec{F}}{m} \]
\[ = \frac{-6.0 \text{ N}}{8.0 \text{ kg}} \]
\[ = -7.5 \text{ m/s}^2 \]
\[ \Delta d = v_2 \Delta t - \frac{1}{2} a(\Delta t)^2 \]
\[ = 0 \text{ m} - \frac{1}{2} \left( -7.5 \text{ m/s}^2 \right)(2.0 \text{ s})^2 \]
\[ = 15 \text{ m} \]
Note that, in this case, the work done is negative. The significance of a negative amount of work done will be clear, later, when we discuss the relationship between work and energy, and the effect of an applied force that opposes an object’s motion.

\[ W = \vec{F} \cdot \Delta \vec{d} \]

\[ = (-60 \text{ N})(15 \text{ m}) \]

\[ = -900 \text{ J, or } -9.0 \times 10^2 \text{ J} \]

Since the horizontal component, \( \vec{F}_h \), is in the same direction as the displacement, the work done by it can be calculated as

\[ W = \vec{F}_h \cdot \Delta \vec{d} \]

\[ = (87 \text{ N})(30 \text{ m})(\cos 0^\circ) \]

\[ = 2610 \text{ J, or } 2.6 \times 10^3 \text{ J (since } \cos 0^\circ = 1) \]

The vertical component, \( \vec{F}_v \), is at a 90° angle with respect to the displacement. To determine the work done by this component, we again use the vector dot product.

\[ W = \vec{F}_v \cdot \Delta \vec{d} \]

\[ = (50 \text{ N})(30 \text{ m}) \cos 90^\circ \]

\[ = 0 \text{ (since } \cos 90^\circ = 0) \]

Of course we could just write it down directly as:
\[ W = \vec{F} \cdot \Delta \vec{d} \]
\[ = |\vec{F}| |\Delta \vec{d}| \cos \theta \]
\[ = (100 \text{ N}) (30 \text{ m}) (0.8660) \]
\[ = 2.6 \times 10^3 \text{ J} \]

REF: K/U OBJ: 4.1 LOC: EMV.01 KEY: FOP 9.1, p.335
MSC: SP

103. ANS:
\[ W = \Delta E_k \]
\[ = E_{k_2} - E_{k_1} \]
\[ = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 \]
\[ = \frac{1}{2} (800 \text{ kg})(30 \text{ m/s})^2 - \frac{1}{2} (800 \text{ kg})(15 \text{ m/s})^2 \]
\[ = 3.6 \times 10^5 \text{ J} - 9.0 \times 10^4 \text{ J} \]
\[ = 2.7 \times 10^5 \text{ J} \]

REF: K/U OBJ: 4.2 LOC: EM1.05 KEY: FOP 9.2, p.341
MSC: SP

104. ANS:
\[ E_{k_1} = \frac{1}{2} m_1 v_1^2 = 16 \text{ J} \]
\[ \therefore v_1 = \sqrt{\frac{2E_{k_1}}{m_1}} \]
\[ = \sqrt{\frac{2(16 \text{ J})}{3.2 \text{ kg}}} \]
\[ = 3.2 \text{ m/s} \]
\[ E_{k_2} = \frac{1}{2} m_2 v_2^2 = 16 \text{ J} \]
\[ \therefore m_2 = \frac{2E_{k_2}}{v_2^2} = \frac{2(16 \text{ J})}{(2.4 \text{ m/s})^2} = 5.6 \text{ kg} \]

105. **ANS:**

For the bullet:

\[ E_k = \frac{1}{2} m v^2 \]
\[ = \frac{1}{2} (0.012 \text{ kg})(400 \text{ m/s})^2 \]
\[ = 960 \text{ J} \]

\[ \therefore \Delta E_k = W \]
\[ = \vec{F} \cdot \Delta \vec{d} \]
\[-960 \text{ J} = \vec{F} \left(3.0 \times 10^{-3} \text{ m}\right) \] (where \( \vec{F} \) is the force exerted by the wood on the bullet)

\[ \vec{F} = -3.2 \times 10^4 \text{ N} \]

Then, by Newton’s Third Law, the force of the bullet on the wood is \( 3.2 \times 10^4 \text{ N} \).

106. **ANS:**

For each interval, \( E_k = \) total area under \( F-d \) graph and \( v = \sqrt{\frac{2E_k}{m}} \).

<table>
<thead>
<tr>
<th>Interval</th>
<th>( E_k )</th>
<th>( v )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 1 m</td>
<td>( \frac{1}{2} (1 \text{ m})(2 \text{ N}) = 1 \text{ J} )</td>
<td>( \sqrt{\frac{(2)(1 \text{ J})}{2.0 \text{ kg}}} = 1.0 \text{ m/s} )</td>
</tr>
<tr>
<td>0 – 2 m</td>
<td>( \frac{1}{2} (2 \text{ m})(4 \text{ N}) = 4 \text{ J} )</td>
<td>( \sqrt{\frac{(2)(4 \text{ J})}{2.0 \text{ kg}}} = 2.0 \text{ m/s} )</td>
</tr>
</tbody>
</table>
### 107. ANS:

Before the collision:

If \( E_{k}(\text{total}) \) at minimum separation is 2.0 J, then

\[
\begin{align*}
0 - 3 \text{ m} & \quad 4 \text{ J} + (1 \text{ m})(4 \text{ N}) = 8 \text{ J} \\
0 - 4 \text{ m} & \quad 8 \text{ J} \\
0 - 5 \text{ m} & \quad 8 \text{ J} + (1 \text{ m})(-1 \text{ N}) = 7 \text{ J}
\end{align*}
\]

\[
\begin{align*}
\sqrt{\frac{8 \text{ J}}{2.0 \text{ kg}}} & = 2.8 \text{ m/s} \\
\sqrt{\frac{7 \text{ J}}{2.0 \text{ kg}}} & = 2.6 \text{ m/s}
\end{align*}
\]

Energy stored during collision = area under the \( F-x \) graph from 0.20 m to 0.05 m

\[
= (10 \text{ N})(0.05 \text{ m}) + (20 \text{ N})(0.05 \text{ m}) + (30 \text{ N})(0.05 \text{ m})
\]

\[
= 0.5 \text{ J} + 1.0 \text{ J} + 1.5 \text{ J}
\]

\[
= 3.0 \text{ J}
\]

\[
\therefore \ E_{\text{before collision}} = E_{\text{at 0.05 m}} + \Delta E
\]

\[
= 4.5 \text{ J} + 3.0 \text{ J}
\]

\[
= 7.5 \text{ J}
\]

\[
\nu = \sqrt{\frac{2E_{k}}{m}}
\]

\[
= \sqrt{\frac{2(7.5 \text{ J})}{1.5 \text{ kg}}}
\]

\[
= 3.2 \text{ m/s}
\]

### 108. ANS:

Before the collision:

\[
E_{k}(\text{total}) = E_{k_1} + E_{k_2}
\]

\[
= \frac{1}{2}(1 \text{ kg})(2.5 \text{ m/s})^2 + \frac{1}{2}(4 \text{ kg})(0.50 \text{ m/s})^2
\]

\[
= 3.13 \text{ J} + 0.50 \text{ J}
\]

\[
= 3.63 \text{ J}
\]

\[
\approx 3.6 \text{ J}
\]

If \( E_{k}(\text{total}) \) at minimum separation is 2.0 J, then
\[ |\Delta E_k(\text{total})| = 3.63 \text{ J} - 2.0 \text{ J} \]
\[ = 1.6 \text{ J} \]
\[ = \text{energy stored during collision} \]
\[ = \text{area under } F\cdot x \text{ graph} \]

\[ \therefore \Delta x(10 \text{ N}) = 1.6 \text{ J} \]
\[ \Delta x = 0.16 \text{ m} \]

\[ \therefore x_0 = 0.25 \text{ m} - 0.16 \text{ m} \]
\[ = 0.09 \text{ m} \]

ANS:
Since both balls are moving before the collision, the specific equations developed earlier do not apply. However, if the collision is viewed from the frame of reference of the 2.0 kg ball before the collision, they do apply. In that frame of reference (i.e., a frame moving to the left at 4.0 m/s),

\[ v_1 = 9.0 \text{ m/s} \text{ and } v_2 = 0 \quad \text{(motion to the right is still taken as positive)} \]

Moving frame of reference

\[
\begin{array}{c}
4.0 \text{ m/s} \\
\downarrow \\
4.0 \text{ kg}
\end{array}

\begin{array}{c}
9.0 \text{ m/s} \\
\rightarrow \\
2.0 \text{ kg}
\end{array}

\begin{array}{c}
\rightarrow \\
\downarrow \\
v_2 = 0
\end{array}

\]

Therefore, \[ v'_1 = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) v_1 \]
\[ = \left( \frac{4.0 \text{ kg} - 2.0 \text{ kg}}{4.0 \text{ kg} + 2.0 \text{ kg}} \right) 9.0 \text{ m/s} \]
\[ = 3.0 \text{ m/s} \]

\[ v'_2 = \left( \frac{2 m_1}{m_1 + m_2} \right) v_1 \]
\[ = \left( \frac{2 (4.0 \text{ kg})}{4.0 \text{ kg} + 2.0 \text{ kg}} \right) 9.0 \text{ m/s} \]
\[ = 12 \text{ m/s} \]
However, both of these velocities are measured in a frame of reference moving to the left at 4.0 m/s. Therefore, in Earth’s frame of reference:

\[ v_1' = -1.0 \text{ m/s} \]

\[ v_2' = 8.0 \text{ m/s} \]

That is, the 4.0 kg ball rebounds to the left at 1.0 m/s, while the 2.0 kg ball rebounds and moves to the right at 8.0 m/s.

(a) \[ v_1' = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) v_1 \]

\[ = \left( \frac{2.4 \text{ kg} - 3.6 \text{ kg}}{2.4 \text{ kg} + 3.6 \text{ kg}} \right) (10 \text{ m/s}) \]

\[ = -2.0 \text{ m/s} \]

\[ v_2' = \left( \frac{2m_1}{m_1 + m_2} \right) v_1 \]

\[ = \left( \frac{2(2.4 \text{ kg})}{2.4 \text{ kg} + 3.6 \text{ kg}} \right) (10 \text{ m/s}) \]

\[ = 8.0 \text{ m/s} \]

\[ E_{k}'(B) = \frac{1}{2} m_2 v_2'^2 \]

\[ = \frac{1}{2} (3.6 \text{ kg})(8.0 \text{ m/s})^2 \]

\[ = 115 \text{ J} \]

\[ E_{k}'(A) = \frac{1}{2} m_1 v_1'^2 \]

\[ = \frac{1}{2} (2.4 \text{ kg})(10 \text{ m/s})^2 \]

\[ = 120 \text{ J} \]
(b) Percent transferred \[= \frac{E_k^r(B)}{E_k^r(A)} \times 100\%\]
\[= \frac{115 \text{ J}}{120 \text{ J}} \times 100\%\]
\[= 96\%\]


111. ANS:
(a) Using conservation of momentum (which always applies to an isolated collision)
\[m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 \vec{v}'_1 + m_2 \vec{v}'_2\]

If \([E]\) is positive,
\[(2.0 \text{ kg})(3.0 \text{ m/s}) + (1.0 \text{ kg})(-2.0 \text{ m/s}) = (2.0 \text{ kg})(1.0 \text{ m/s}) + (1.0 \text{ kg})(v'_2)\]
\[6.0 \text{ kg \cdot m/s} - 2.0 \text{ kg \cdot m/s} - 2.0 \text{ kg \cdot m/s} = 1.0 \text{ kg} \cdot v'_2\]
\[v'_2 = 2.0 \text{ m/s}\]
\[\vec{v}'_2 = 2.0 \text{ m/s} [E]\]

(b) Before the collision:
\[E_k^r\text{(total)} = E_k^r(1) + E_k^r(2)\]
\[= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2\]
\[= \frac{1}{2} (2.0 \text{ kg})(3.0 \text{ m/s})^2 + \frac{1}{2} (1.0 \text{ kg})(-2.0 \text{ m/s})^2\]
\[= 9.0 \text{ J} + 2.0 \text{ J}\]
\[= 11.0 \text{ J}\]

After the collision:
\[E_k'^r\text{(total)} = E_k'^r(1) + E_k'^r(2)\]
\[= \frac{1}{2} m_1 v'_1^2 + \frac{1}{2} m_2 v'_2^2\]
\[= \frac{1}{2} (2.0 \text{ kg})(1.0 \text{ m/s})^2 + \frac{1}{2} (1.0 \text{ kg})(2.0 \text{ m/s})^2\]
\[- 1.0 \text{ J} + 2.0 \text{ J}\]
\[= 3.0 \text{ J}\]

\[\triangle E_k\text{(total)} = 3.0 \text{ J} - 11.0 \text{ J}\]
\[= - 8.0 \text{ J}\]
Therefore, the collision is not elastic.

ANS:

(a) Assume a frame of reference moving with the 200 g glider. In that frame \( v_1 = 150 \text{ cm/s} \) and \( v_2 = 0 \).

Then, \( v'_1 = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) v_1 \)

\[
= \left( \frac{300 \text{ g} - 200 \text{ g}}{300 \text{ g} + 200 \text{ g}} \right) (150 \text{ cm/s}) \\
= 30 \text{ cm/s}
\]

\( v'_2 = \left( \frac{2m_1}{m_1 + m_2} \right) v_1 \)

\[
= \left( \frac{2(300 \text{ g})}{300 \text{ g} + 200 \text{ g}} \right) (150 \text{ cm/s}) \\
= 30 \text{ cm/s}
\]

In the original frame of reference, these velocities are

\( v'_1 = -70 \text{ cm/s} \)

\( v'_2 = 80 \text{ cm/s} \)

(b) Using conservation of momentum,

\[
m_1 \vec{v}_1 + m_2 \vec{v}_2 = (m_1 + m_2) \vec{v}_{12}
\]

\[
(300 \text{ kg})(50 \text{ cm/s}) + (200 \text{ g})(-100 \text{ cm/s}) = (500 \text{ g}) \vec{v}_{12}
\]

\[
\vec{v}_{12} = \frac{15 \text{ 000 g} \cdot \text{ cm/s} - 20 \text{ 000 g} \cdot \text{ cm/s}}{500 \text{ g}}
\]

\[
= -10 \text{ cm/s}
\]
\[ E_k(\text{before}) = E_k(1) + E_k(2) \]
\[ = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \]
\[ = \frac{1}{2} (0.300 \text{ kg})(0.50 \text{ m/s})^2 + \frac{1}{2} (0.200 \text{ kg})(-1.0 \text{ m/s})^2 \]
\[ = 3.75 \times 10^{-3} \text{ J} + 1.0 \times 10^{-2} \text{ J} \]
\[ = 1.38 \times 10^{-1} \text{ J} \]

\[ E_k(\text{stack together}) = \frac{1}{2} (m_1 + m_2)v_{12}^2 \]
\[ = \frac{1}{2} (0.500 \text{ kg})(-0.10 \text{ m/s})^2 \]
\[ = 2.5 \times 10^{-3} \text{ J} \]

\[ \Delta E_k = E_k(\text{stack together}) - E_k(\text{before}) \]
\[ = 2.5 \times 10^{-3} \text{ J} - 1.38 \times 10^{-1} \text{ J} \]
\[ - 0.03 \times 10^{-1} \text{ J} - 1.38 \times 10^{-1} \text{ J} \]
\[ = -1.35 \times 10^{-1} \text{ J} \]

113. **Ans:**
(a) Assume a frame of reference moving at 2.0 m/s [right]. In that frame \( \vec{v}_1 = 4.0 \text{ m/s [right]} \) and \( \vec{v}_2 = 0 \).

Then, \( v_1' = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) v_1 \)

\[ = \left( \frac{6.0 \text{ kg} - 2.0 \text{ kg}}{6.0 \text{ kg} + 2.0 \text{ kg}} \right) (4.0 \text{ m/s}) \]
\[ = 2.0 \text{ m/s} \]
\[ v_2' = \left( \frac{2m_1}{m_1 + m_2} \right) v_1 \]
\[ = \left( \frac{12.0 \text{ kg}}{6.0 \text{ kg} + 2.0 \text{ kg}} \right) (4.0 \text{ m/s}) \]
\[ = 6.0 \text{ m/s} \]

Then, in Earth’s frame of reference:
\[ \vec{v}_1' = 4.0 \text{ m/s} \text{ [right]} \]
\[ \vec{v}_2' = 8.0 \text{ m/s} \text{ [right]} \]

(b) Before the collision:
\[ E_k(\text{total}) = E_k(1) + E_k(2) \]
\[ = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2 \]
\[ = \frac{1}{2} (6.0 \text{ kg})(6.0 \text{ m/s})^2 + \frac{1}{2} (2.0 \text{ kg})(2.0 \text{ m/s})^2 \]
\[ = 108 \text{ J} + 4 \text{ J} \]
\[ = 112 \text{ J} \]

At minimum separation:
\[ m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 \vec{v}_0 + m_2 \vec{v}_0 \]
\[ (6.0 \text{ kg})(6.0 \text{ m/s}) + (2.0 \text{ kg})(2.0 \text{ m/s}) = (6.0 \text{ kg} + 2.0 \text{ kg}) \vec{v}_0 \]
\[ \vec{v}_0 = 5.0 \text{ m/s} \]

Then, \[ E_k(\text{total}) = E_k(1) + E_k(2) \]
\[ = \frac{1}{2} (m_1 + m_2) \vec{v}_0^2 \]
\[ = \frac{1}{2} (8.0 \text{ kg})(5.0 \text{ m/s})^2 \]
\[ = 100 \text{ J} \]

\[ \therefore \text{ Energy stored} = -\Delta E_k \]

and \[ \Delta E_k = 100 \text{ J} - 112 \text{ J} = -12 \text{ J} \]
Energy stored = \(- \Delta E_k\)

\[= - (-12 \text{ J})\]

\[= 12 \text{ J}\]

114. ANS:
(a) The collision is elastic since its \( F-d \) graph is a function (i.e., only one value of \( F \) for any value of \( d \)).

(b) \( v'_1 = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) v_1 \)

\[= \left( \frac{8.0 \text{ kg}}{16.0 \text{ kg}} \right) (0.80 \text{ m/s})\]

\[= 0.40 \text{ m/s}\]

\( v'_2 = \left( \frac{2m_1}{m_1 + m_2} \right) v_1 \)

\[= \left( \frac{24.0 \text{ kg}}{16.0 \text{ kg}} \right) (0.80 \text{ m/s})\]

\[= 1.2 \text{ m/s}\]

(c) At minimum separation, \( \Delta \vec{p} = 0 \)

\( m_1 \vec{v}_1 + m_2 \vec{v}_2 = (m_1 + m_2) \vec{v}_0 \)

\( (12.0 \text{ kg})(0.80 \text{ m/s}) + (4.0 \text{ kg})(0 \text{ m/s}) = (16.0 \text{ kg}) \vec{v}_0 \)

\( \vec{v}_0 = 0.60 \text{ m/s}\)

\( E_k(\text{total}) = E_k(1) + E_k(2) \)

\[= \frac{1}{2} (m_1 + m_2)(v_0)^2\]

\[= \frac{1}{2} (16.0 \text{ kg})(0.60 \text{ m/s})^2\]

\[= 2.88 \text{ J}\]

(d) Before the collision:
\[ E_k(\text{total}) = E_k(1) + E_k(2) \]
\[ = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \]
\[ = \frac{1}{2} (12.0 \text{ kg})(0.30 \text{ m/s})^2 + 0 \text{ J} \]
\[ = 3.84 \text{ J} \]

Therefore, energy stored = \( -\Delta E_k \).

\[ \Delta E_k = E_{k2} - E_{k1} \]
\[ = 2.88 \text{ J} - 3.84 \text{ J} \]
\[ = -0.96 \text{ J} \]

= area under \( F \cdot d \) graph from \( x = 0.10 \text{ m} \) to \( x_0 \).

Energy stored = \( -\Delta E_k \)
\[ = -(-0.96 \text{ J}) \]
\[ = 0.96 \text{ J} \]

To find \( x_0 \):

Area from \( x = 0.10 \text{ m} \) to \( x = 0.06 \text{ m} \) is \( \frac{1}{2} (0.04 \text{ m})(24 \text{ N}) = 0.48 \text{ J} \).

Therefore, remaining area = \( 0.96 \text{ J} - 0.48 \text{ J} \)
\[ = 0.48 \text{ J} \]

which is a rectangle of height 24 J and width \( \Delta x \).

\( (24 \text{ N})(\Delta x) = 0.48 \text{ J} \)
\[ \Delta x = 0.02 \text{ m} \]
\[ \therefore x_0 = (0.06 - 0.02) \text{ m} \]
\[ = 0.04 \text{ m} \]
But, $\Delta d = v\Delta t$

$$= (20 \text{ m/s}) \left( 3.6 \times 10^5 \text{ s} \right)$$

$$= 7.2 \times 10^4 \text{ m}$$

$\therefore W = \left( 5.4 \times 10^4 \text{ N} \right) \left( 7.2 \times 10^4 \text{ N} \right)$

$$= 3.9 \times 10^9 \text{ J}$$

(b) $W = \vec{F} \cdot \Delta \vec{d}$

$$= F \Delta d \cos \theta$$

$$= (500 \text{ N})(120 \text{ m})(\cos 15^\circ)$$

$$= 5.8 \times 10^4 \text{ J}$$

(c) $W = \vec{F}_g \cdot \Delta \vec{d}$

$$= F_g (5.0 \text{ m})$$

But, $F_g = mg$

$$= \left( 30 \text{ kg} \right) \left( 9.8 \text{ N/kg} \right)$$

$$= 294 \text{ N}$$

$\therefore W = (294 \text{ N})(5.0 \text{ m})$

$$= 1.5 \times 10^3 \text{ J}$$

(d) $W = \vec{F}_e \cdot \Delta \vec{d}$

$$= F_e \Delta d \cos \theta$$

$$= \left( 12 \text{ kg} \right) \left( 9.8 \text{ N/kg} \right) (30 \text{ m})(\cos 90^\circ)$$

$$= 0 \text{ J}$$

(e) $W = \vec{F} \cdot \Delta \vec{d}$

$$= \text{area under } \vec{F} \cdot \Delta \vec{d} \text{ graph from 0 to 8 m}$$

$$= \frac{1}{2} (2 \text{ m})(2 \text{ N}) + (3 \text{ m})(2 \text{ N}) + (2 \text{ m}) \left( \frac{2 \text{ N} + 4 \text{ N}}{2} \right) + (1 \text{ m})(4 \text{ N})$$

$$= 2 \text{ J} + 6 \text{ J} + 6 \text{ J} + 4 \text{ J}$$

$$= 18 \text{ J}$$
116. ANS:
(a) \( W = \Delta E_x \)
\[
\begin{align*}
&= E'_x - E_x \\
&= \frac{1}{2} m v'_x^2 - \frac{1}{2} m v^2 \\
&= \frac{1}{2} \left( 40 \text{ kg} \right) \left( 20 \text{ m/s} \right)^2 - \frac{1}{2} \left( 40 \text{ kg} \right) \left( 10 \text{ m/s} \right)^2 \\
&= 8.0 \times 10^3 \text{ J} - 2.0 \times 10^3 \text{ J} \\
&= 6.0 \times 10^3 \text{ J}
\end{align*}
\]

(b) \( W = \Delta E_k \)
\[
\begin{align*}
&= E'_k - E_k \\
&= \frac{1}{2} m v'_k^2 - 2.0 \times 10^3 \text{ J} \\
&= \frac{1}{2} \left( 40 \text{ kg} \right) \left( 5.0 \text{ m/s} \right)^2 - 2.0 \times 10^3 \text{ J} \\
&= 0.50 \times 10^3 \text{ J} - 2.0 \times 10^3 \text{ J} \\
&= -1.5 \times 10^3 \text{ J}
\end{align*}
\]

117. ANS:
For work done on the pile by the driver:
\[
W = \overrightarrow{F} \cdot \Delta \overrightarrow{d}
\]
\[
= \left( 3.5 \times 10^5 \text{ N} \right) (0.50 \text{ m})
\]
\[
= 1.75 \times 10^5 \text{ J}
\]

Therefore, for work done on the piledriver by Earth:
\[
W = \overrightarrow{F} \cdot \Delta \overrightarrow{h}
\]
\[
1.75 \times 10^5 \text{ J} = F \cdot \Delta h
\]
But, $F_g = mg$

$$= (1500 \text{ kg})(9.8 \text{ N/kg})$$

$$= 1.47 \times 10^4 \text{ N}$$

$$1.75 \times 10^5 \text{ J} = \left(1.47 \times 10^4 \text{ N}\right)(\Delta h)$$

$$\therefore \Delta h = 11.9 \text{ m or } 12 \text{ m}$$

REF: K/U OBJ: 4.4 LOC: EM1.05 KEY: FOP 9.9, p.368
MSC: P

118. ANS:

(a) $W = \Delta E_k$

$$1.25 \times 10^4 \text{ J} = \frac{1}{2}mv^2 - 0 \text{ J}$$

$$= \frac{1}{2} m (50 \text{ m/s})^2$$

$$2 \left(1.25 \times 10^4 \text{ J}\right)$$

$$m = \frac{2 \left(1.25 \times 10^4 \text{ J}\right)}{(50 \text{ m/s})^2}$$

$$= 10 \text{ kg}$$

(b) $\vec{F} \cdot \Delta \vec{d} = \frac{1}{2}mv^2$

$$\vec{F}(5.0 \text{ m}) = \frac{1}{2} (10 \text{ kg})(50 \text{ m/s})^2$$

$$\vec{F} = 2.5 \times 10^3 \text{ N}$$

REF: K/U OBJ: 4.4 LOC: EM1.05 KEY: FOP 9.9, p.369
MSC: P

119. ANS:
(a) \[ P_1 = P_2 \]

\[ m_1 v_1 = m_2 v_2 \]

\[ v_2 = \frac{m_1 v_1}{m_2} \]

\[ = \frac{(4.0 \text{ kg})(20 \text{ m/s})}{(10.0 \text{ kg})} \]

\[ = 8.0 \text{ m/s} \]

(b) \[ E_k(1) = \frac{1}{2} m_1 v_1^2 \]

\[ = \frac{1}{2} (4.0 \text{ kg})(20 \text{ m/s})^2 \]

\[ = 8.0 \times 10^2 \text{ J} \]

\[ E_k(2) = \frac{1}{2} m_2 v_2^2 \]

\[ = \frac{1}{2} (10.0 \text{ kg})(8.0 \text{ m/s})^2 \]

\[ = 3.2 \times 10^2 \text{ J} \]
Kinetic energy gained by pellet = work done on pellet in barrel

\[ \frac{1}{2} m v^2 = \text{area under } F-d \text{ graph from 0 to 1.0 m} \]

\[ = \left( \frac{1.2 \times 10^2 \text{ N} + 0.6 \times 10^2 \text{ N}}{2} \right) (1.0 \text{ m}) \]

\[ - 0.9 \times 10^2 \text{ J} \]

\[ v = \sqrt{\frac{2 \left( 0.9 \times 10^2 \text{ J} \right)}{5.0 \times 10^{-3} \text{ kg}}} \]

\[ = \sqrt{3.6 \times 10^4 \text{ J/kg}} \]

\[ = 1.9 \times 10^2 \text{ m/s} \]

ANS:

For such a collision,

\[ v'_1 = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) v_1 \]

\[ = \left( \frac{0.80 \text{ kg} - 0.40 \text{ kg}}{0.80 \text{ kg} + 0.40 \text{ kg}} \right) (8.0 \text{ m/s}) \]

\[ = \left( \frac{0.40 \text{ kg}}{1.20 \text{ kg}} \right) (8.0 \text{ m/s}) \]

\[ = 2.7 \text{ m/s} \]

\[ v'_2 = \left( \frac{2 m_1}{m_1 + m_2} \right) v_1 \]

\[ = \left( \frac{2 \times 0.80 \text{ kg}}{1.20 \text{ kg}} \right) (8.0 \text{ m/s}) \]

\[ = 10.7 \text{ m/s} \]

ANS:
(a) For such a collision, $\Delta \vec{p} = 0$

$$\vec{p}_1 + \vec{p}_2 = \vec{p}'_1 + \vec{p}'_2$$

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 \vec{v}'_1 + m_2 \vec{v}'_2$$

$$(3000 \text{ kg})(5.0 \text{ m/s}) + m_2(2.0 \text{ m/s}) = (3000 \text{ kg})(3.0 \text{ m/s}) + m_2(6.0 \text{ m/s})$$

$$(4.0 \text{ m/s}) m_2 = 6.0 \times 10^3 \text{ kg} \cdot \text{m/s}$$

$$m_2 = 1.5 \times 10^3 \text{ kg}$$

(b) $E'_k (\text{total}) = E'_{k_1} + E'_{k_2}$

$$= \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

$$= \frac{1}{2} (3000 \text{ kg})(5.0 \text{ m/s})^2 + \frac{1}{2} (1500 \text{ kg})(2.0 \text{ m/s})^2$$

$$= 3.75 \times 10^4 \text{ J} + 0.30 \times 10^4 \text{ J}$$

$$= 4.05 \times 10^4 \text{ J or } 4.1 \times 10^4 \text{ J}$$

$E'_k (\text{total}) = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$

$$= \frac{1}{2} (3000 \text{ kg})(3.0 \text{ m/s})^2 + \frac{1}{2} (1500 \text{ kg})(6.0 \text{ m/s})^2$$

$$= 1.35 \times 10^4 \text{ J} + 2.7 \times 10^4 \text{ J}$$

$$= 4.05 \times 10^4 \text{ J or } 4.1 \times 10^4 \text{ J}$$

(c) Since $E'_k (\text{total}) = E_k (\text{total})$, the collision is elastic.
\[ v' = \left( \frac{2m_1}{m_1 + m_2} \right) v_1 \]
\[ = \left( \frac{2(1.0 \text{ kg})}{1.0 \text{ kg} + 0.50 \text{ kg}} \right) (0.24 \text{ m/s}) \]
\[ = 0.32 \text{ m/s} \]

(b) At minimum separation \( v_1 = v_2 = v_0 \) and \( \Delta \vec{p} = \vec{0} \).
\[ m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 \vec{v}_0 + m_2 \vec{v}_0 \]
\[ (1.0 \text{ kg})(0.24 \text{ m/s}) + (0.50 \text{ kg})(0 \text{ m/s}) = (1.50 \text{ kg})(v_0) \]
\[ v_0 = \frac{0.24 \text{ kg} \cdot \text{m/s}}{1.50 \text{ kg}} \]
\[ = 0.16 \text{ m/s} \]

(c) Then, \( E_k(\text{minimum}) = E_{k_1} + E_{k_2} \)
\[ = \frac{1}{2} m_1 v_0^2 + \frac{1}{2} m_2 v_0^2 \]
\[ = \frac{1}{2} (1.0 \text{ kg})(0.16 \text{ m/s})^2 + \frac{1}{2} (0.05 \text{ kg})(0.16 \text{ m/s})^2 \]
\[ = 1.28 \times 10^{-2} \text{ J} + 0.64 \times 10^{-2} \text{ J} \]
\[ = 1.92 \times 10^{-2} \text{ J} \text{ or } 1.9 \times 10^{-2} \text{ J} \]

(d) \( \Delta E_p = - \Delta E_k \)
\[ = - \left( E_k(\text{minimum}) - E_k(\text{before}) \right) \]
where \( E_k(\text{before}) = E_{k_1} + E_{k_2} \)
\[ = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \]
\[ = \frac{1}{2} (1.0 \text{ kg})(0.24 \text{ m/s})^2 + 0 \text{ J} \]
\[ = 2.88 \times 10^{-2} \text{ J} \]
\[ \Delta E_p = -\left( 1.92 \times 10^{-2} \text{ J} - 2.88 \times 10^{-2} \text{ J} \right) \]
\[ = 9.6 \times 10^{-3} \text{ J} \]

124. ANS:
(a) In order that the equations developed for the special case apply, assume a frame of reference moving to the left at 0.15 m/s. In that frame,

\[ \nu_1 = 0.55 \text{ m/s and } \nu_2 = 0 \]

\[ \nu'_1 = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) \nu \]

\[ = \left( \frac{0.30 \text{ kg} - 0.80 \text{ kg}}{0.30 \text{ kg} + 0.80 \text{ kg}} \right) (0.55 \text{ m/s}) \]
\[ = 0.25 \text{ m/s} \]

\[ \nu'_2 = \left( \frac{2m_1}{m_1 + m_2} \right) \nu \]
\[ = \left( \frac{2(0.30 \text{ kg})}{(0.30 \text{ kg} + 0.80 \text{ kg})} \right) (0.55 \text{ m/s}) \]
\[ = 0.30 \text{ m/s} \]

Therefore, in Earth’s frame,
\[ \nu'_1 = -0.40 \text{ m/s} \]
\[ \nu'_2 = 0.15 \text{ m/s} \]

(b) At minimum separation, \( \nu_1 = \nu_2 = \nu_0 \) and \( \Delta \vec{p} = 0 \).

\[ \vec{p}_{\text{total}}(\text{before}) = \vec{p}_{\text{total}}(\text{minimum separation}) \]
\[ m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 \vec{v}_0 + m_2 \vec{v}_0 \]
\[ (0.30 \text{ kg})(0.40 \text{ m/s}) + (0.80 \text{ kg})(-0.15 \text{ m/s}) = (1.10 \text{ kg}) \nu_0 \]

\[ \nu_0 = \frac{0.12 \text{ kg} \cdot \text{m/s} - 0.12 \text{ kg} \cdot \text{m/s}}{1.10 \text{ kg}} \]
\[ = 0 \text{ m/s} \]
\[ E_k(\text{minimum separation}) = 0 \]

(c) The “missing” energy is stored as elastic potential energy in the spring bumper.

\[ \text{REF: K/U, C OBJ: 5.3 LOC: EM1.03 KEY: FOP 9.9, p.370 MSC: P} \]

125. ANS:

At the moment of impact, the geometry is as shown:

After the collision, the paths and speeds of the balls are:
\[ v_1 = v_2 = v \] by symmetry

Since momentum is conserved, in the \( x \)-direction,
\[ m(20 \text{ m/s}) = mv \cos 30^\circ + mv \cos 30^\circ + mv \]
\[ 20 = v \frac{\sqrt{3}}{2} + v \frac{\sqrt{3}}{2} + v \]
\[ 20 = v + v \sqrt{3} \] (1)

Also, since kinetic energy is conserved,
\[ \frac{1}{2} m(20 \text{ m/s})^2 = \frac{1}{2} mv^2 + \frac{1}{2} mv^2 + \frac{1}{2} mv^2 \]
\[ 400 = 2v^2 + v^2 \] (2)

From (1), \[ v = \frac{20 - v}{\sqrt{3}} \]

Substituting into (2)
But $V = 20 \text{ m/s}$ was the initial velocity, so that $V = -4 \text{ m/s}$; the cue ball bounces back.

(a) In the interaction, $\Delta p = 0$

\[ \vec{p}_{\text{total \ (before)}} = \vec{p}_{\text{total \ (after)}} \]

\[ m\vec{v} + 0 = m\vec{v'} + M\vec{V} \]

\[
\begin{align*}
(4.0 \times 10^{-3} \text{ kg})(5.0 \times 10^2 \text{ m/s}) &= (4.0 \times 10^{-3} \text{ kg})(1.0 \times 10^2 \text{ m/s}) + (2.0 \text{ kg})V_2 \\
\end{align*}
\]

\[ V = \frac{2.0 \text{ kg} \cdot \text{m/s} - 4.0 \text{ kg} \cdot \text{m/s}}{2.0 \text{ kg}} = 0.80 \text{ m/s} \]

(b) \[ E_k = \frac{1}{2}Mv^2 \]

\[ = \frac{1}{2} (2.0 \text{ kg})(0.80 \text{ m/s})^2 = 0.64 \text{ J} \]
(c) $W = F \Delta d = \Delta E_k$

$F = \frac{\Delta E_k}{\Delta d}$

$=-0.64 \text{ J}$

$= \frac{0.40 \text{ m}}{1.6 \text{ N}}$

$=-1.6 \text{ N}$

(d) $\Delta E_k = E_k' - E_k$

$= \frac{1}{2} m \overline{v}^2 - \frac{1}{2} m \overline{v}^2$

$= \frac{1}{2} \left( 4.0 \times 10^{-3} \text{ kg} \right) \left[ \left( 1.0 \times 10^2 \text{ m/s} \right)^2 - \left( 5.0 \times 10^2 \text{ m/s} \right)^2 \right]$

$= \left( 2.0 \times 10^{-3} \text{ kg} \right) \left( -24 \times 10^4 \text{ m}^2 / \text{s}^2 \right)$

$=-4.8 \times 10^2 \text{ J}$

(e) Kinetic energy is not conserved but is lost to heat, sound, and the permanent deformation of both the block and (to some degree) the bullet.

REF: K/U, C
OBJ: 5.3
LOC: EM1.03
KEY: FOP 9.9, p.371
MSC: P

127. ANS:

Since momentum is conserved, $\Delta \overline{p} = 0$ and $\overline{p}_T$ (before) = 0.

$\overline{p}_T$ (after) = $m \overline{v} + 5m \overline{V} = 0$

$\overline{v} = \frac{-5m \overline{V}}{m}$

$\overline{v} = -5 \overline{V}$

$\overline{v} = 5 \overline{V}$
\[ E_{k_1} = \frac{1}{2} m v_1^2 \]
\[ \therefore \frac{E_{k_1}}{E_{k_2}} = \frac{1}{2} m v_1^2 \]
\[ = \frac{1}{2} m (5V)^2 \]
\[ = \frac{1}{2} (25m) V^2 \]
\[ = 5 \]

\[ \therefore \frac{E_{k_1}}{5 + 1} = \frac{5}{6} \]

128. ANS:

(a) \( E_k \) (before) = \( \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \)
\[ = \frac{1}{2} \left( 5.0 \text{ kg} \right) \left( 6.0 \text{ m/s} \right)^2 + 0 \text{ J} \]
\[ = 0.90 \text{ J} \]

\( E_k \) (after) = 0.90 J since the collision is elastic because the F-d graph is a function of d only.

(b) At minimum separation:
\[ \vec{p}_{\text{total}} \] (before) = \[ \vec{p}_{\text{total}} \] (minimum separation)
and \( v_1 = v_2 = v_0 \)

\[ m_1 v_1 + m_2 v_2 = m_1 v_0 + m_2 v_0 \]
\[ \left( 5.0 \text{ kg} \right) \left( 6.0 \text{ m/s} \right) + 0 \text{ J} = \left( 7.5 \text{ kg} \right) \left( v_0 \right) \]
\[ v_0 = 0.40 \text{ m/s} \]

(c) At minimum separation:
\[ E_k(\text{minimum}) = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \]
\[ = \frac{1}{2} \left( 5.0 \text{ kg} \right) \left( 0.40 \text{ m/s} \right)^2 + \frac{1}{2} \left( 2.5 \text{ kg} \right) \left( 0.40 \text{ m/s} \right)^2 \]
\[ = 0.40 \text{ J} + 0.20 \text{ J} \]
\[ = 0.60 \text{ J} \]

(d) Energy stored = \( \Delta E_k(\text{total}) \)
\[ = -(E_k(\text{min})) - E_k(\text{before}) \]
\[ = -0.60 \text{ J} + 0.90 \text{ J} \]
\[ = 0.30 \text{ J} \]

(e) Minimum separation distance = point on \( F-d \) graph where area under graph is 0.30 J

From 0.030 m to 0.020 m,
Energy stored = \( (0.010 \text{ m})(15 \text{ N}) \)
\[ = 0.15 \text{ J} \]

From 0.020 m to 0.01 m, need 0.15 J more energy.

Therefore, width of the rectangle whose area is 0.15 J is \( \frac{0.15 \text{ J}}{30 \text{ N}} = 0.0050 \text{ m} \).

Minimum separation distance = 0.020 m - 0.0050 m
\[ = 0.015 \text{ m} \]

(f) At \( x = 0.015 \text{ m}, \) reading from the graph \( F = 30 \text{ N} \).

\[ W = F \Delta d \]

and \( F = \frac{F_g}{m} = mg \)
\[ = \left( 12 \text{ kg} \right) \left( 9.8 \text{ N/kg} \right) \]
\[ = 118 \text{ N} \]

Therefore, \( W = (118 \text{ N})(3.0 \text{ m}) \)
\[ = 354 \text{ J}, \text{ or } 3.5 \times 10^2 \text{ J} \]
Calculating the elastic potential energy stored in the spring.

At \( x = -0.30 \text{ m} \):
\[
E_x = \frac{1}{2} k x^2
\]
\[
= \frac{1}{2} (100 \text{ N/m})(-0.30 \text{ m})^2
\]
\[
= 4.5 \text{ J}
\]

At \( x = -0.10 \text{ m} \):
\[
E_x = \frac{1}{2} k x^2
\]
\[
= \frac{1}{2} (100 \text{ N/m})(-0.10 \text{ m})^2
\]
\[
= 0.50 \text{ J}
\]

Therefore, as the spring expands from a compression of 0.30 m to 0.10 m, the change in stored potential energy is given by
\[
\Delta E_x = E_{x_2} - E_{x_1}
\]
\[
= 0.50 \text{ J} - 4.5 \text{ J}
\]
\[
= -4.0 \text{ J}
\]

This change in elastic potential energy for the spring represents a loss in stored energy, and the amount of energy lost is equal to the kinetic energy gained by the block.
\[
\Delta E_k = \frac{1}{2} m \nu^2
\]
\[
4.0 \text{ J} = \frac{1}{2} (0.5 \text{ kg}) \nu^2
\]
\[
\nu^2 = 16(\text{m/s})^2
\]
\[
\nu = 4.0 \text{ m/s}
\]

Note: In this type of energy transfer, the total energy of the system of the block and spring, at all times, is given by
\[
E_{\text{total}} = E_k(\text{block}) + E_x(\text{spring})
\]
\[
= \text{constant}
\]
\[
= 4.5 \text{ J in this case}
\]

In practice, frictional effects will usually cause some of the total energy to dissipate as heat. These frictional forces usually act in the opposite direction to the motion, so that they do negative work.
The 2.0 kg cart rebounds and moves to the left at 1.0 m/s, while the 4.0 kg cart moves to the right at 2.0 m/s.

(b) Let \( v_0 \) be the velocity at minimum separation.

Since \( \Delta p_{\text{total}} = 0 \),
\[
\begin{align*}
 p_{\text{total}}(\text{before}) &= p_{\text{total}}(\text{at minimum separation}) \\
 m_1 v_1 + m_2 v_2 &= m_1 v_0 + m_2 v_0 = (m_1 + m_2) v_0 \\
 (2.0 \text{ kg})(3.0 \text{ m/s}) + 0 \text{ kg \cdot m/s} &= (6.0 \text{ kg}) v_0 \\
 v_0 &= 1.0 \text{ m/s}
\end{align*}
\]

At minimum separation, both carts are moving to the right at 1.0 m/s.

(c) \( \Delta E_k = E_k(\text{at minimum separation}) - E_k(\text{before}) \)
\[
\begin{align*}
 &= \left( \frac{1}{2} m_1 v_0^2 \right) - \left( \frac{1}{2} m_1 v_1^2 + 0 \right), \\
 &\quad \text{since } m_2 \text{ is initially at rest} \\
 &= \frac{1}{2} (6.0 \text{ kg})(1.0 \text{ m/s})^2 - \frac{1}{2} (2.0 \text{ kg})(3.0 \text{ m/s})^2 \\
 &= 3.0 \text{ J} - 9.0 \text{ J} \\
 &= -6.0 \text{ J}
\end{align*}
\]
Note that the change in kinetic energy is negative, representing a loss in total kinetic energy at minimum separation.

(d) At minimum separation, the kinetic energy lost has been transformed into elastic potential energy stored in the compressed spring, so that the total energy of the system remains constant.
\[ \Delta E_k = -\Delta E_e = 6.0 \text{ J} \]

\[ E_e = \frac{1}{2} kx^2 \]

\[ 6.0 \text{ J} = \frac{1}{2} (1200 \text{ N/m}) x^2 \]

\[ x = \pm 0.10 \text{ m} \]

But, since the spring is compressed, \( x = -0.10 \text{ m} \).

(e) The minimum separation distance, \( L_0 \) is the normal length of the spring, \( L_0 \), plus its deformation at minimum separation, \( x_0 \).
\[ L_0 = L + x_0 \]
\[ = 0.25 \text{ m} + (-0.10 \text{ m}) \]
\[ = 0.15 \text{ m} \]

REF: K/U OBJ: 5.3, 4.5 LOC: EM1.03 KEY: FOP 10.3, p.378
MSC: SP

132. ANS:
\[ E_k = 2 \left( \frac{1}{2} m \nu^2 \right) \]
\[ = m \nu^2 \]
\[ = (2.5 \text{ kg})(3.0 \text{ m/s})^2 \]
\[ = 22.5 \text{ J} \]

\[ |E_k| = |E_e| \]

Then, \( k = \frac{2E_e}{x^2} \)
\[ = \frac{(2)(22.5 \text{ J})}{(0.10 \text{ cm})^2} \]
\[ = 4500 \text{ N/m} \]

REF: K/U OBJ: 4.5 LOC: EM1.05 KEY: FOP 10.3, p.379
133. ANS:
(a) $E_k(\text{total}) = E_{k1} + E_{k2}$
\[= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2\]
\[= \frac{1}{2} \left(4.5 \, \text{kg}\right)(2.0 \, \text{m/s})^2 + \frac{1}{2} \left(1.0 \, \text{kg}\right)(4.0 \, \text{m/s})^2\]
\[= 9.0 \, \text{J} + 8.0 \, \text{J}\]
\[= 17 \, \text{J}\]

(b) At minimum separation, momentum is conserved.
\[\begin{align*}
- & \quad \rightarrow + \\
\vec{m}_1 \vec{v}_1 + \vec{m}_2 \vec{v}_2 &= \vec{m}_1 \vec{v}_0 + \vec{m}_2 \vec{v}_0 \\
&= \left(\vec{m}_1 + \vec{m}_2\right) \vec{v}_0 \\
\left(4.5 \, \text{kg}\right)(2.0 \, \text{m/s}) + \left(1.0 \, \text{kg}\right)(-4.0 \, \text{m/s}) &= 5.5 \, \text{kg} \, \vec{v}_0 \\
\vec{v}_0 &= \frac{5.0 \, \text{kg} \cdot \text{m/s}}{5.5 \, \text{kg}} \\
\vec{v}_0 &= 0.91 \, \text{m/s} \, [\text{right}] \\
\end{align*}\]

(c) $E_{k0} = (m_1 + m_2) v_0^2$
\[= \frac{1}{2} \left(5.5 \, \text{kg}\right)(0.91 \, \text{m/s})^2\]
\[= 2.3 \, \text{J}\]

(d) $E_e = - \Delta E_k$
\[= - \left(E_{k0} - E_{k1}\right) \\
= -(2.3 \, \text{J} - 17 \, \text{J}) \\
= 14.7 \, \text{J}\]

Then, $x = \sqrt{\frac{2E_e}{k}}$
\[= \sqrt{\frac{2(14.7 \, \text{J})}{900 \, \text{N/m}}}\]
\[= 0.18 \, \text{m}\]
134. ANS:

(a) Momentum is conserved, and \( \vec{v}_1 = \vec{v}_2 = \vec{v}_0 \) at minimum separation.

\[
\begin{align*}
\vec{v}_1 &= \vec{v}_2 = \vec{v}_0 \\
(m_1 \vec{v}_1 + m_2 \vec{v}_2) &= (m_1 \vec{v}_0 + m_2 \vec{v}_0) \\
- (m_1 + m_2) \vec{v}_0
\end{align*}
\]

\[
(50 \text{ g})(-4.0 \text{ cm/s}) + (30 \text{ g})(20 \text{ cm/s}) = (80 \text{ g})\vec{v}_0
\]

\[
\vec{v}_0 = \frac{400 \text{ g} \cdot \text{cm/s}}{80 \text{ g}}
\]

\[
\vec{v}_0 = 5.0 \text{ cm/s} \text{ [right]}
\]

(b) \( \Delta E_e = - \Delta E_k \)

\[
\begin{align*}
\Delta E_e &= - \Delta E_k \\
&= - \left( E_{k0} - E_{k1} \right) \\
&= - \left( \frac{1}{2} \left( m_1 + m_2 \right) (v_0)^2 - \frac{1}{2} m_1 v_1^2 - \frac{1}{2} m_2 v_2^2 \right) \\
&= - \left( \frac{1}{2} (80 \text{ g})(5.0 \text{ cm/s})^2 - \frac{1}{2} (50 \text{ g})(-4.0 \text{ cm/s})^2 - \frac{1}{2} (30 \text{ g})(20 \text{ cm/s})^2 \right) \\
&= 5400 \text{ g} \cdot \text{cm}^2 / \text{s}^2 \\
&= 5.4 \times 10^{-4} \text{ J}
\end{align*}
\]

(c) \( E_e = \frac{1}{2} k x^2 \)

\[
\begin{align*}
k &= - \frac{2E_e}{x^2} \\
&= - \frac{2 \times 5.4 \times 10^{-4} \text{ J}}{\left(1.5 \times 10^{-2} \text{ m}\right)^2} \\
&= 4.8 \text{ N/m}
\end{align*}
\]
(a) The total energy of the system consisting of the ball and Earth remains constant at all points on the 1 to 2 to 3 path, and is equal to the sum of the initial kinetic energy and the gravitational potential energy. Assuming that the zero level of $E_g$ is the surface of Earth, $E_{total} = E_k + E_g$ at all points.

Therefore, at point 1:

$$E_{total_1} = E_k_1 + E_g_1$$

$$= \frac{1}{2} m v_1^2 + mg \Delta h_1$$

$$= \frac{1}{2} (0.25 \text{ kg})(16 \text{ m/s})^2 + (0.25 \text{ kg})(9.8 \text{ N/kg})(18 \text{ m})$$

$$= 32 \text{ J} + 44 \text{ J}$$

$$= 76 \text{ J}$$

At the instant when the ball reaches its maximum height, $E_{g_2} = 0$, and all of its energy is in the form of gravitational potential.

$$E_{total_2} = E_k_2 + E_g_2 = E_{total_1} = 76 \text{ J}$$

$$- 0 + mg \Delta h_2$$

$$\Delta h_2 = \frac{76 \text{ J}}{(0.25 \text{ kg})(9.8 \text{ N/kg})}$$

$$= 31 \text{ m}$$

(b) When the ball reaches ground level, $E_{g_3} = 0$, and all of its energy is kinetic.

$$E_{total_3} = E_k_3 + E_g_3 = 76 \text{ J}$$

$$= \frac{1}{2} m v_3^2 + 0$$

$$v_3^2 = \frac{2(76 \text{ N})}{0.25 \text{ kg}}$$

$$= 608 \text{ (m/s)}^2$$

$$v_3 = -24.7 \text{ m/s}, \text{ or } -25 \text{ m/s}$$

Identical results are obtained if $E_g = 0$ is chosen at the roof level of the building, but don’t forget that $E_{g_3}$ will then be negative at ground level.

REF: K/U       OBJ: 4.4       LOC: EM1.05       KEY: FOP 10.4, p.382
MSC: SP

136. ANS:
(a) The total energy of the roller coaster-Earth system remains constant and is equal to the gravitational potential energy at A (take \( E_g = 0 \) at ground level).
\[
E_{\text{total}} = E_k + E_g = E_g \text{ (at A)} \quad \text{since} \quad E_g \text{ (at A)} = 0
\]
\[
= mg\Delta h_A
\]
\[
= (1000 \text{ kg})(9.8 \text{ N/kg})(9.5 \text{ m})
\]
\[
= 9.3 \times 10^4 \text{ J}
\]
The kinetic energy, and hence the velocity, will be a maximum at the point where the gravitational potential energy is a minimum (i.e., at point D).
\[
E_g \text{ (at D)} = mg\Delta h_D
\]
\[
= (1000 \text{ kg})(9.8 \text{ N/kg})(0.50 \text{ m})
\]
\[
= 4.9 \times 10^3 \text{ J}
\]
Therefore, \( E_k \text{ (at D)} = E_{\text{total}} - E_g \text{ (at D)} \)
\[
= 9.3 \times 10^4 \text{ J} - 4.9 \times 10^3 \text{ J}
\]
\[
= 8.8 \times 10^4 \text{ J}
\]
\[
\nu = \sqrt{\frac{2E_k}{m}}
\]
\[
= \sqrt{\frac{2(8.8 \times 10^4 \text{ J})}{1000 \text{ kg}}}
\]
\[
= 13.3 \text{ m/s, or 13 m/s}
\]
(b) \( E_g \text{ (at E)} = mg\Delta h_E \)
\[
= (1000 \text{ kg})(9.8 \text{ N/kg})(5.5 \text{ m})
\]
\[
= 5.4 \times 10^4 \text{ J}
\]
Therefore, \( E_k \text{ (at E)} = E_{\text{total}} - E_g \text{ (at E)} \)
\[
= 9.3 \times 10^4 \text{ J} - 5.4 \times 10^4 \text{ J}
\]
\[
= 3.9 \times 10^4 \text{ J}
\]
\[
\begin{align*}
\nu &= \sqrt{\frac{2E_k}{m}} \\
&= \sqrt{\frac{2(3.9 \times 10^4 \text{ J})}{1000 \text{ kg}}} \\
&= 8.83 \text{ m/s, or } 8.8 \text{ m/s}
\end{align*}
\]

(c) The brakes do work on the roller coaster to reduce its kinetic energy from \(3.9 \times 10^4 \text{ J}\) to zero.

work done = change in \(E_k\)

\[
F \Delta d = E_{k_f} - E_{k_i}
\]

\[
= 0 \text{ J} - 3.9 \times 10^4 \text{ J}
\]

\[
F = \frac{-3.9 \times 10^4 \text{ J}}{5.0 \text{ m}}
\]

\[
= -7.8 \times 10^3 \text{ N}
\]

The negative sign indicates a force in the opposite direction to motion, as a frictional braking force would be.

REF: K/U, MC OBJ: 4.4 LOC: EM1.05 KEY: FOP 10.4, p.383 MSC: SP

137. ANS:

After the spring is released, the total energy of the spring-marble-Earth system remains constant. Let the height of the marble, at release, be the zero level of gravitational potential energy.

Just before release:

\[
E_{\text{total}} = E_e + E_g + E_k
\]

\[
= \frac{1}{2} kx^2 + mg\Delta h + \frac{1}{2}mv^2
\]

\[
= \frac{1}{2} \left( \frac{F}{x} \right) x^2 + 0 \text{ J} + 0 \text{ J}
\]

\[
= \frac{1}{2} \left( \frac{-50 \text{ N}}{-0.10 \text{ m}} \right)(-0.10 \text{ m})^2
\]

\[
= 2.5 \text{ J}
\]

Then, as the marble reaches its maximum height,
\[ E_{\text{total}} = E_e + E_k + E_g \]

\[ 2.5 \text{ J} = 0 \text{ J} + 0 \text{ J} + mg\Delta h \]

\[ \Delta h = \frac{2.5 \text{ J}}{(0.020 \text{ kg})(9.8 \text{ m/s})} \]

\[ = 12.8 \text{ m} \]

Note that this distance, being positive, is above the starting point. The spring will have expanded 0.10 m to its normal length, so that the marble is 12.7 m above the top of the now-uncompressed spring.

ANS:

(a) \[ \Delta E_g = - \Delta E_k \]

\[ mg\Delta h = - \left[ E_{k2} - E_{k1} \right] \]

where \( E_{k2} = \frac{1}{2}mv_2^2 \) and \( E_{k1} = 0 \)

\[ (70 \text{ kg})(9.8 \text{ N/kg})(-12 \text{ m}) = - \frac{1}{2} (70 \text{ kg})(v_2^2) \]

\[ -8232 \text{ J} = - (35 \text{ kg})v_2^2 \]

\[ v_2 = 15 \text{ m/s} \]

(b) Again, \[ mg\Delta h = - \left[ E_{k2} - E_{k1} \right] \]

But \( E_{k1} \neq 0 \).

\[ (70 \text{ kg})(9.8 \text{ N/kg})(-12 \text{ m}) = - \left( \frac{1}{2} (70 \text{ kg}) (v_2)^2 - \frac{1}{2} (70 \text{ kg})(5.0 \text{ m/s})^2 \right) \]

\[ -8232 \text{ J} = - (35 \text{ kg})(v_2)^2 + 875 \text{ J} \]

\[ v_2^2 = \frac{8232 \text{ J} + 875 \text{ J}}{35 \text{ kg}} \]

\[ = 260.2 \text{ J/kg} \]

\[ v_2 = 16 \text{ m/s} \]
\[ \Delta h = (2.0 \text{ m} - 0.85 \text{ m}) = 1.5 \text{ m} \]

\[ -\Delta E_k = \Delta E_g \]

\[ - \left[ E_{k_2} - E_{k_1} \right] = mg\Delta h \quad \text{where } E_{k_2} = 0 \]

\[ \frac{1}{2} m v_1^2 = mg\Delta h \]

\[ v_1^2 = 2g\Delta h \]

\[ = 2(9.8 \text{ N/kg})(1.15 \text{ m}) \]

\[ = 22.54 \text{ N} \cdot \text{m/kg} \]

\[ v_1 = 4.7 \text{ m/s} \]

140. ANS:

(a) \[ E_e = \frac{1}{2} k\chi^2 \]

\[ = \frac{1}{2}(800 \text{ N/m})(0.22 \text{ m})^2 \]

\[ = 19.36 \text{ J, or } 19 \text{ J} \]

(b) \[ \Delta E_k = -\Delta E_e \]

\[ E_{k_2} - E_{k_1} = - \left( E_{e_2} - E_{e_1} \right) \quad \text{where } E_{k_1} = 0 \text{ and } E_{e_1} = 0 \]

\[ \frac{1}{2} m v_2^2 = \frac{1}{2} k\chi_1^2 \]

\[ v_2 = \sqrt{\frac{k\chi_1^2}{m}} \]

\[ = \sqrt{\frac{(800 \text{ N/m})(0.22 \text{ m})^2}{2.0 \text{ kg}}} \]

\[ = 4.4 \text{ m/s} \]

(c) \[ \Delta E_k = \Delta E_g \]

\[ - \left[ E_{k_2} - E_{k_1} \right] = E_{e_2} - E_{e_1} \quad \text{where } E_{k_2} = 0 \text{ and } E_{e_1} = 0 \]
\[ \frac{1}{2} m v_1^2 = mg \Delta h \]

\[ \Delta h = \frac{v_1^2}{2g} = \frac{(4.4 \text{ m/s})^2}{2(9.8 \text{ N/kg})} = 1.0 \text{ m} \]

141. **ANS:**

\[-\Delta E_e = \Delta E_z \]

\[- \left[ E_{e_2} - E_{e_1} \right] = E_{z_2} - E_{z_1} \]

where \( E_{e_2} = 0 \) and \( E_{z_1} = 0 \)

\[ \frac{1}{2} k x_1^2 = mg \Delta h \]

\[ x_1 = \sqrt{\frac{2mg \Delta h}{k}} = \sqrt{\frac{(2)(0.050 \text{ kg})(9.8 \text{ N/kg})(3.1 \text{ m})}{120 \text{ N/m}}} = 0.16 \text{ m, or 16 cm} \]

142. **ANS:**

\( r_1 = r_e = 6.4 \times 10^6 \text{ m} \)

\( r_2 = 2r_e \)

\[ \Delta E_e = E_{e_2} - E_{e_1} = \left( \frac{-GM_e m}{r_2} \right) - \left( \frac{-GM_e m}{r_1} \right) = GM_e m \left( \frac{r_2 - r_1}{r_2 r_1} \right) \]
But, $\Delta E_g = -\Delta E_k = \frac{1}{2}mv^2$

\[\therefore \quad \frac{1}{2}mv^2 = GM_m m \left(\frac{r_2-r_1}{r_2r_1}\right)\]

\[v = \sqrt{\frac{2GM_m(r_2-r_1)}{r_2r_1}}\]

\[= \sqrt{\left(\frac{2\left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right)\left(5.98 \times 10^{24} \text{ kg}\right)}{6.4 \times 10^6 \text{ m}}\right)}\]

\[= 7.9 \times 10^3 \text{ m/s}\]

REF: K/U OBJ: 6.3 LOC: EM1.06 KEY: FOP 10.5, p.390

MSC: P

143. ANS:

\[r_2 = r_1 + \left(100 \times 10^3 \text{ m}\right)\]

\[= \left(6.37 \times 10^6 \text{ m}\right) + \left(0.10 \times 10^6 \text{ m}\right)\]

\[= 6.47 \times 10^6 \text{ m}\]

$\Delta E_g = GM_m m \left(\frac{r_2-r_1}{r_2r_1}\right)$

\[= \left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right)\left(5.98 \times 10^{24} \text{ kg}\right)\left(1.0 \times 10^5 \text{ m}\right)\]

\[= 1.0 \times 10^6 \text{ J}\]

$\Delta E_k = mg\Delta h$

\[= \left(1.0 \text{ kg}\right)\left(9.8 \text{ N/kg}\right)\left(1.0 \times 10^5 \text{ m}\right)\]

\[= 0.98 \times 10^6 \text{ J}\]
144. ANS:

(a) A synchronous satellite remains in the same relative position above Earth because it has a period of 24 h, the same as that of Earth’s rotation on its axis.

As for any satellite, 
\[ F_c = F_g \]

\[
\frac{4\pi^2 m_s r_o}{T^2} = \frac{GM_E m_s}{r_c^2}
\]

\[
r_o = \sqrt[3]{\frac{GM_E T^2}{4\pi^2}}
\]

\[
= \sqrt[3]{\frac{\left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right) \left(5.98 \times 10^{24} \text{ kg}\right) \left(8.64 \times 10^4 \text{ s}\right)^2}{4\pi^2}}
\]

\[
= 3 \sqrt[3]{\frac{75.4 \times 10^{21} \text{ m}^3}{4\pi^2}}
\]

\[
= 4.22 \times 10^7 \text{ m}
\]

This radius represents an altitude of \(3.6 \times 10^4 \text{ km}\) above Earth’s surface.

(b) At the surface of Earth:

\[
E_g = \frac{GM_E m_s}{r_E} - \left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right) \left(5.98 \times 10^{24} \text{ kg}\right) \left(5.00 \times 10^2 \text{ kg}\right)
\]

\[
= - \frac{5.98 \times 10^{24} \text{ kg} \left(5.00 \times 10^2 \text{ kg}\right)}{6.4 \times 10^6 \text{ m}}
\]

\[
= - 3.12 \times 10^{10} \text{ J}
\]

(c) The total energy of a satellite in orbit, bound to the Earth, is given by
\[ E_{\text{total}} = E_k + E_g \]
\[ = \frac{1}{2} m_s v_o^2 - \frac{GM_s m_s}{r_o} \]
\[ = - \frac{1}{2} \frac{GM_s m_s}{r_o} \]
\[ = - \frac{1}{2} \left( 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \right) \left( 5.98 \times 10^{24} \text{ kg} \right) \left( 5.00 \times 10^2 \text{ kg} \right) \]
\[ = - \frac{1}{2} \frac{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2}{4.22 \times 10^7 \text{ m}} \]
\[ = - 2.36 \times 10^9 \text{ J} \]

(d) Work done \( \Delta E_{\text{total}} \)
\[ = E_{\text{total}} \text{ (in orbit)} - E_{\text{total}} \text{ (on Earth)} \]
\[ = - 2.36 \times 10^9 \text{ J} - \left( -3.12 \times 10^{10} \text{ J} \right) \]
\[ = 2.88 \times 10^{10} \text{ J} \]

(e) in orbit, \( E_{\text{total}} = - 2.36 \times 10^9 \text{ J} \)
for escape, \( E_{\text{total}} \geq 0 \)

Therefore, in order to escape, the satellite must acquire at least \( 2.36 \times 10^9 \) J of additional energy.

REF: K/U, MC OBJ: 6.3 LOC: EM1.06 KEY: FOP 10.6, p.395
MSC: SP
145. ANS:
(a) \( r_o = 6.37 \times 10^6 \text{ m} + 0.50 \times 10^6 \text{ m} \)
\[ = 6.87 \times 10^6 \text{ m} \]

For an orbiting satellite,
\[ E_{\text{total}} = - \frac{1}{2} \frac{GM_s m_s}{r_o} \]
\[ = - \frac{1}{2} \left( 6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \right) \left( 5.98 \times 10^{24} \text{ kg} \right) \left( 2.0 \times 10^3 \text{ kg} \right) \]
\[ = - \frac{6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}}{6.87 \times 10^6 \text{ m}} \]
\[ = - 5.80 \times 10^{10} \text{ J} \]
At rest on Earth:

\[ E_{\text{total}} = -\frac{GM_e m_s}{r_e} \]

\[ = \left( 6.67 \times 10^{-11} \, \text{N} \cdot \text{m}^2 / \text{kg}^2 \right) \left( 5.98 \times 10^{24} \, \text{kg} \right) \left( 2.0 \times 10^3 \, \text{kg} \right) \]

\[ = -1.25 \times 10^{11} \, \text{J} \]

\[ \therefore \Delta E_{\text{total}} = E_2 - E_1 \]

\[ = -5.80 \times 10^{10} \, \text{J} \]

\[ = 6.7 \times 10^{10} \, \text{J} \]

(b) In orbit:

\[ E_{\text{total}} = -5.80 \times 10^{10} \, \text{J} \]

\[ \therefore \Delta E_{\text{escape}} = 5.80 \times 10^{10} \, \text{J} \]

REF: K/U OBJ: 6.3 LOC: EM1.07 KEY: FOP 10.6, p.396

MSC: P

146. ANS:

(a) \[ \Delta E_k = -\Delta E_e \]

\[ \frac{1}{2} m v^2 = \frac{1}{2} k x^2 \]

\[ v = \sqrt{\frac{k x^2}{m}} \]

\[ = \sqrt{\frac{(500 \, \text{N/m}) \left( 5.0 \times 10^{-2} \, \text{m} \right)^2}{1.0 \times 10^{-2} \, \text{kg}}} \]

\[ = 11.1 \, \text{m/s}, \text{ or } 11 \, \text{m/s} \]

(b) Work done by retarding force \( \vec{F} \cdot \Delta \vec{d} \)

\[ = (-0.80 \, \text{N})(0.25 \, \text{m}) \]

\[ = -0.20 \, \text{J} \]
\[ \Delta E_e = \frac{1}{2} k x^2 \]

\[ = \frac{1}{2} (500 \text{ N/m})(5.0 \times 10^{-2} \text{ m})^2 \]

\[ = 0.625 \text{ J} \]

\[ \therefore \Delta E_e = 0.625 \text{ J} - 0.20 \text{ J} \]

\[ = 0.425 \text{ J} \]

\[ v = \sqrt{\frac{2E_k}{m}} \]

\[ = \sqrt{\frac{(2)(0.425 \text{ J})}{0.010 \text{ kg}}} \]

\[ = 9.2 \text{ m/s} \]

147. ANS:

(a) \[ E_e = \frac{1}{2} k x^2 \]

\[ = \frac{1}{2} (100 \text{ N/m})(0.20 \text{ m})^2 \]

\[ = 2.0 \text{ J} \]

(b) \[ \Delta E_k = \Delta E_e = 2.0 \text{ J} \]

\[ v = \sqrt{\frac{2E_k}{m}} \]

\[ = \sqrt{\frac{(2)(2.0 \text{ J})}{0.50 \text{ kg}}} \]

\[ = 2.83 \text{ m/s}, \text{ or } 2.8 \text{ m/s} \]

(c) Momentum is conserved in the impact:
\( m\nu = (m + M)\nu \)
\[ \nu = \left( \frac{m + M}{m} \right)\nu \]
\[ = \left( \frac{0.500 \text{ kg}}{0.010 \text{ kg}} \right) (2.83 \text{ m/s}) \]
\[ = 142 \text{ m/s, or } 1.4 \times 10^2 \text{ m/s} \]

(d) \( E_k = \frac{1}{2} mv^2 \)
\[ = \frac{1}{2} \left( 0.010 \text{ kg} \right) \left( 1.42 \times 10^2 \text{ m/s} \right)^2 \]
\[ = 1.0 \times 10^2 \text{ J} \]

\[ \Delta E_{\text{total}} = 2.0 \text{ J} - 100 \text{ J} = -98 \text{ J} \]
and represents energy lost to heat, sound, and the permanent deformation of
the bullet and the block.

REF: K/U OBJ: 5.3 LOC: EM1.03 KEY: FOP 10.8, p.401
MSC: P

148. ANS:

\[ \begin{array}{c}
\begin{array}{c}
2.4 \text{ kg} \\
\end{array}
\end{array} \]
\[ \begin{array}{c}
\begin{array}{c}
1.5 \text{ m/s} \\
\end{array}
\end{array} \]
\[ \begin{array}{c}
\begin{array}{c}
3.6 \text{ kg} \\
\end{array}
\end{array} \]

(a) \( E_{\text{total}} = \frac{1}{2} m_1 v_1^2 \)
\[ = \frac{1}{2} \left( 2.4 \text{ kg} \right) \left( 1.5 \text{ m/s} \right)^2 \]
\[ = 2.7 \text{ J} \]

(b) At \( x_0 \), \( v_1 = v_2 = v_0 \) and momentum is conserved.
\[ m_1 v_1 = (m_1 + m_2) v_0 \]

\[ v_0 = \frac{m_1 v_1}{m_1 + m_2} \]

\[ = \frac{(2.4 \text{ kg})(1.5 \text{ m/s})}{(2.4 \text{ kg} + 3.6 \text{ kg})} \]

\[ = 0.60 \text{ m/s} \]

(c) \[ \Delta E_k = E_k (\text{minimum separation}) - E_k (\text{before}) \]

\[ = \frac{1}{2} (m_1 + m_2) (v_0)^2 - 2.7 \text{ J} \]

\[ = \frac{1}{2} (6.0 \text{ kg})(0.60 \text{ m/s})^2 - 2.7 \text{ J} \]

\[ = 1.08 \text{ J} - 2.7 \text{ J} \]

\[ = -1.58 \text{ J}, \text{ or } -1.6 \text{ J} \]

(d) \[ E_e = \frac{1}{2} k x^2 \text{ and } \Delta E_e = -\Delta E_k = 1.6 \text{ J} \]

\[ k = \frac{2E_e}{x^2} \]

\[ = \frac{(2)(1.58 \text{ J})}{(0.12 \text{ m})^2} \]

\[ = 219 \text{ N/m, or } 2.2 \times 10^2 \text{ N/m} \]

Momentum is conserved, and \( \vec{P}_{\text{total}} = 0 \).
\[ m_1 \vec{v}_1 + m_2 \vec{v}_2 = 0 \]
\[(1.2 \text{ kg})(v'_1) + (4.8 \text{ kg})(2.0 \text{ m/s}) = 0 \]
\[ v'_1 = -3.0 \text{ m/s} \]

\[ E_k'(\text{after release}) = E_{k1}' + E_{k2}' \]
\[ = \frac{1}{2} m_1 v'_1^2 + \frac{1}{2} m_2 v'_2^2 \]
\[ = \frac{1}{2} (1.2 \text{ kg})(-8.0 \text{ m/s})^2 + \frac{1}{2} (4.8 \text{ kg})(2.0 \text{ m/s})^2 \]
\[ = 38.4 \text{ J} + 9.6 \text{ J} \]
\[ = 48 \text{ J} \]

Therefore, for the potential energy in spring before release:

\[ \Delta E_e = -\Delta E_k \]
\[ \frac{1}{2} kx^2 = 48 \text{ J} \]

\[ x = \sqrt{\frac{2(48 \text{ J})}{2400 \text{ N/m}}} \]
\[ = 0.20 \text{ m} \]

REF: K/U OBJ: 5.3 LOC: EM1.03 KEY: FOP 10.8, p.401
MSC: P

ANS: 150.

\[ \Delta E_e = -\Delta E_g \]
\[ \frac{1}{2} kx^2 = mg(\Delta h + x) \]
\[ \frac{1}{2} (1200 \text{ N/m})x^2 = (3.0 \text{ kg})(9.8 \text{ N/kg})(0.80 + x) \]
\[ 600x^2 = 23.5 + 29.4x \]
\[ 6000x^2 - 294x - 235 = 0 \]
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ = \frac{294 \pm \sqrt{(294)^2 + 4(6000)(235)}}{12000} \]

\[ = \frac{294 \pm 2393}{12000} \]

\[ = 0.22 \text{ m or } -0.17 \text{ m} \]

But the negative solution is inadmissible.

---

151. ANS:

\[ mg(\Delta k_1 + x_1) = \frac{1}{2} k x_1^2 \]

\[ (60 \text{ kg})(9.8 \text{ N/kg})(11 \text{ m}) = \frac{1}{2} k (1.0 \text{ m})^2 \]

\[ k = 1.3 \times 10^4 \text{ N/m} \]

\[ mg(\Delta k_2 + x_2) = \frac{1}{2} k x_2^2 \]

\[ 588(20 + x_2) = 6500 x_2^2 \]

\[ 6500 x_2^2 - 588 x_2 - 11760 = 0 \]

\[ x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ = \frac{588 \pm \sqrt{(588)^2 - 4(6500)(11760)}}{13000} \]

\[ = \frac{588 \pm 17496}{13000} \]

\[ = 1.4 \text{ m} \]

The negative solution is inadmissible.

---

152. ANS:
At point 1 where the spring is extended to 28.5 cm,
\( x_1 = 28.5 \text{ cm} - 15.0 \text{ cm} = 13.5 \text{ cm} \)

\[ F_1 = kx_1 \]
\[ = (70 \text{ N/m})(0.135 \text{ cm}) \]
\[ = 9.45 \text{ N} \]

\[ F_e = mg \]
\[ = (0.500 \text{ kg})(9.8 \text{ N/kg}) \]
\[ = 4.9 \text{ N} \]
\[ \therefore a = \frac{F_{\text{net}}}{m} \]
\[ = \frac{(9.45 - 4.9) \text{ N}}{0.500 \text{ kg}} \]
\[ = 9.1 \text{ m/s}^2 \]

(b) At any point in its path, the total energy of the system is given by
\[ E_T = E_k + E_g + E_e. \]

If we assume that the reference level of \( E_g \) is at the lowest point, point 1, when \( L = 28.5 \text{ cm} \), then at that point
\[ E_{T1} = E_{k1} + E_{g1} + E_{e1} \]
\[ = 0 + 0 - \frac{1}{2} kx_1^2 \]
\[ = \frac{1}{2} (70 \text{ N/m})(0.135 \text{ m})^2 \]
\[ = 0.638 \text{ J} \]

Thus, at point 2, when \( L = 22.0 \text{ cm} \)
\[ x_2 = 22.0 \text{ cm} - 15.0 \text{ cm} = 7.0 \text{ cm} \]

\[ 0.638 \text{ J} = E_{k2} + E_{g2} + E_{e2} \]
\[ = \frac{1}{2} m v_2^2 + mgh_2 + \frac{1}{2} kx_2^2 \]
\[ 0.638 J = \frac{1}{2} (0.500 \text{ kg})v_2^2 + \left[ (0.500 \text{ kg})(9.8 \text{ N/m})(0.285 \text{ m} - 0.220 \text{ m}) \right] + \frac{1}{2} (70 \text{ N/m})(0.070 \text{ m})^2 \]
\[ 0.638 J = (0.250 \text{ kg})v_2^2 + 0.319 J + 0.172 J \]
\[ v_2^2 = 0.588 \text{ J/kg} \]
\[ v_2 = 0.77 \text{ m/s} \]

(c) At point 3, when \( v = 0 \) at the highest point, \( \Delta h \), above point 1
\[ x_3 = 28.5 \text{ cm} - \Delta y - 15.0 \text{ cm} \]
\[ = (13.5 \text{ cm}) - \Delta y \]
Therefore, the length of the spring is 28.5 cm – 13 cm = 15.5 cm, stretched by only 0.5 cm.

\[ 0.638 \text{ J} = E_{k_3} + E_{e_3} + E_{s_3} \]
\[ = 0 + mg\Delta h + \frac{1}{2} k (0.135 - \Delta h)^2 \]
\[ 0.638 = (0.500 \text{ kg})(9.8 \text{ N/m})\Delta h + \frac{1}{2} (70 \text{ N/m})(0.135 - \Delta h)^2 \]
\[ 638 = 4900\Delta h + (35000)(0.0182 - 0.27\Delta h + \Delta h^2) \]
\[ 638 = 4900\Delta h + 638 - 9450\Delta h^2 + 35000\Delta h^2 \]
\[ 35000\Delta h^2 - 4550\Delta h^2 = 0 \]
\[ \Delta h = 0, \text{ or } \Delta h = \frac{4550}{35000} \]
\[ = 0.13 \text{ m} \]

Therefore, the length of the spring is 28.5 cm – 13 cm = 15.5 cm, stretched by only 0.5 cm.

REF: K/U OBJ: 4.5 LOC: EM1.05 KEY: FOP 10.8, p.402
MSC: P

153. ANS:

(a) \( k_A = \frac{F_A}{x_A} \)
\[ = \frac{10 \text{ N}}{0.25 \text{ m}} \]
\[ = 40 \text{ N/m} \]

\( k_B = \frac{F_B}{x_B} \)
\[ = \frac{2.5 \text{ N}}{0.25 \text{ m}} \]
\[ = 10 \text{ N/m} \]
\[
W = \Delta E_{e_A} + \Delta E_{e_B} \\
= \frac{1}{2} k_A x_A^2 + \frac{1}{2} k_B x_B^2 \\
= \frac{1}{2} (40 \text{ N/m})(0.50 \text{ m})^2 + \frac{1}{2} (10 \text{ N/m})(0.50 \text{ m})^2 \\
= 5.0 \text{ J} + 1.25 \text{ J} \\
= 6.25 \text{ J}, \text{ or } 6.3 \text{ J}
\]

(b) At original position, all energy is \(E_k\)

\[
v = \sqrt{\frac{2E_k}{m}} \\
= \sqrt{\frac{(2)(6.25 \text{ J})}{(1.0 \text{ kg})}} \\
= 35 \text{ m/s}
\]

(c) To duplicate A and C means to store the same energy at the same compression.

\[
k = \frac{2E_e}{x^2} \\
= \frac{2(6.25 \text{ J})}{(0.50 \text{ m})^2} \\
= 50 \text{ N/m}
\]

REF: K/U OBJ: 4.5 LOC: EM1.05 KEY: FOP 10.8, p.402

MSC: P

154. ANS:
(a) \( E_{\varepsilon_1} = mg \Delta h \)
\[ = (1.0 \text{ kg})(9.8 \text{ N/kg})(5.0 \text{ m}) \]
\[ = 49 \text{ J} \]

But, \( E_{\varepsilon_2} = E_{\varepsilon_1} \)
\[ \therefore v_2 = \sqrt{\frac{2E_{\varepsilon_1}}{m}} \]
\[ = \sqrt{\frac{2(49 \text{ J})}{1.0 \text{ kg}}} \]
\[ = 9.9 \text{ m/s} \]

(b) Centripetal force is supplied by \( F_g \), down, and tension, up (see diagram).

\[
F_c = T - F_g
\]
\[
\frac{mv^2}{r} = T - mg
\]
\[
T = mg + \frac{mv^2}{r}
\]
\[
= (1.0 \text{ kg})(9.8 \text{ N/kg}) + \frac{(1.0 \text{ kg})(9.9 \text{ m/s})^2}{(5.0 \text{ m})}
\]
\[ = 9.8 \text{ N} + 19.6 \text{ N} \]
\[ = 29.4 \text{ N}, \text{ or } 29 \text{ N} \]
\[ \frac{\Delta h}{\Delta d} = \sin 30^\circ \]
\[ \Delta h = \Delta d \sin 30^\circ \]

Using conservation of energy:
\[ \frac{1}{2} m v_f^2 = m g \Delta h + W_f \Delta d \]

\[ \frac{1}{2} (5.0 \text{ kg})(6.0 \text{ m/s})^2 = (5.0 \text{ kg})(9.8 \text{ N/kg})(\Delta d \sin 30^\circ) + (4.0 \text{ N})(\Delta d) \]

\[ 90 \text{ J} = 24.5 \text{ J/m} \Delta d + 4.0 \text{ J/m} \Delta d \]
\[ \Delta d = 3.16 \text{ m}, \text{ or } 3.2 \text{ m} \]

Then, \[ \Delta E_k = m g \Delta h \]
\[ = (5.0 \text{ kg})(9.8 \text{ N/kg})(3.16 \text{ m} \sin 30^\circ) \]
\[ = 77 \text{ J} \]

The other 135 J went to friction.

ANS: (a) Centripetal force at B is supplied by \( F_c \) and the force of the track pushing down. The minimum force of the track is zero.
\[ F_c = F_g \]
\[ \frac{m v^2}{r} = mg \]
\[ v = \sqrt{rg} \]
\[ = \sqrt{(10 \text{ m})(9.8 \text{ N/kg})} \]
\[ = 9.9 \text{ m/s} \]
(b) Using conservation of energy:

\[ E_{\varepsilon_A} = E_{\varepsilon_B} + E_{\varepsilon_B} \]

\[ m g \Delta h_A = \frac{1}{2} m v_B^2 + m g \Delta h_B \]

\[ \Delta h_A = \frac{\frac{1}{2} v_B^2 + g \Delta h_B}{g} \]

\[ = \frac{\frac{1}{2} (9.9 \text{ m/s})^2 + (9.8 \text{ N/kg})(20 \text{ m})}{9.8 \text{ N/kg}} \]

\[ = \frac{49 \text{ (m/s)}^2 + 196 \text{ (m/s)}^2}{9.8 \text{ N/kg}} \]

\[ = 25 \text{ m} \]

(c) The mass is not a factor, since it divides out of the equation in (b) regardless of its value.

REF: K/U, MC OBJ: 4.4 LOC: EM1.05 KEY: FOP 10.8, p.402

MSC: P

157. ANS:

At the time the 15.0 kg box (1) hits the floor, both boxes will have the same speed (if the string doesn’t stretch).

\[ E_{\varepsilon_1} = E_{\varepsilon_1} + E_{\varepsilon_2} + E_{\varepsilon_2} \]

\[ m_1 g \Delta h_1 = \frac{1}{2} m_1 v^2 + m_2 g \Delta h_2 + \frac{1}{2} m_2 v^2 \]

\[(15.0 \text{ kg})(9.8 \text{ N/kg})(2.5 \text{ m}) = \frac{1}{2} (15.0 \text{ kg}) v^2 + (5.0 \text{ kg})(9.8 \text{ N/kg})(2.5 \text{ m}) + \frac{1}{2} (5.0 \text{ kg}) v^2 \]

\[367.5 J = 7.5 \text{ kg} v^2 + 122.5 J + 2.5 \text{ kg} v^2 \]

\[ v^2 = \frac{245 J}{10 \text{ kg}} \]

\[ = 24.5 \text{ J/kg} \]

\[ v = 4.9 \text{ m/s} \]

REF: K/U OBJ: 4.4 LOC: EM1.05 KEY: FOP 10.8, p.403

MSC: P

158. ANS:
(a) \( E_g = mg\Delta h \)
\[
= (1.0 \text{ kg})(9.8 \text{ N/kg})(-0.50 \text{ m})
\]
\[
= -4.9 \text{ J (reference X)}
\]

(b) \( E_g = mg\Delta h \)
\[
= (1.0 \text{ kg})(9.8 \text{ N/kg})(0 \text{ m})
\]
\[
= 0 \text{ (reference Y)}
\]

(c) (i) At point A, directly below Y:
\( E_k = \frac{1}{2}mv^2 \)
\[
= mg\Delta h
\]
\[
= (1.0 \text{ kg})(9.8 \text{ N/kg})(0.80 \text{ m} - 0.50 \text{ m})
\]
\[
= 29 \text{ J}
\]

(ii) At that same point, A:
\[
v = \sqrt{\frac{2E_k}{m}}
\]
\[
= \sqrt{\frac{(2)(2.9 \text{ J})}{(1.0 \text{ kg})}}
\]
\[
= 2.4 \text{ m/s}
\]

(iii) At point C, on a horizontal plane with Z and Y:
\( E_g = mg\Delta h = 0 \text{ (reference Y)} \)
\( \Delta h = 0 \)

C and Y are on same level, \( E_g = 0 \).

(d) To swing completely around Y, with the minimum speed at the top, tension \( T = 0 \)

Then, total energy at that point, with respect to Y is
\[ E_T = E_k + E_g \]
\[ = \frac{1}{2}mv^2 + mg\Delta h_2 \]
\[ = \frac{1}{2} \left( 1.0 \text{ kg} \right) (1.71 \text{ m/s})^2 + \left( 1.0 \text{ kg} \right) (9.8 \text{ N/kg}) (0.30 \text{ m}) \]
\[ = 1.47 \text{ J} + 2.94 \text{ J} \]
\[ = 4.41 \text{ J} \]

Therefore, height above Y at which the ball has 4.41 J of potential energy and no kinetic energy is

\[ \Delta h = \frac{E_T}{mg} \]
\[ = \frac{4.41 \text{ J}}{\left( 1.0 \text{ kg} \right) (9.8 \text{ N/kg})} \]
\[ = 0.45 \text{ m} \]

159. \textbf{ANS:}

(a) \[ E_g \text{ (before drop)} = mg\Delta h \]
\[ = m (9.8 \text{ N/kg}) (1.0 \text{ m}) \]
\[ = 9.8m \text{ J} \]

\[ E_g \text{ (before bounce)} = 9.8m \text{ J} \]

\[ E_g \text{ (after bounce)} = (0.90)(9.8m)J \]

\[ \therefore E_g \text{ (after rise)} = (0.90)(9.8m)J \]

\[ \therefore \Delta h = \frac{E_g}{mg} \]
\[ = \frac{(0.90)(9.8m) \text{ J}}{m (9.8 \text{ N/kg})} \]
\[ = 0.90 \text{ m} \]

(b) The total distance is \[ d = 1.0 \text{ m} + 2 \left( 0.9 + 0.9^2 + 0.9^3 + \ldots \right) \text{ m} \] which has the form of an infinite geometric progression and may be written, in terms of its sum, as
\[ d = 1.0 \text{ m} \left[ 1 + 2S_\omega \right] \quad \text{where} \quad S_\omega = \frac{a}{1 - r} \]

\[ - 1.0 \text{ m} \left[ 1 + 2 \left( \frac{0.9}{1 - 0.9} \right) \right] \]

\[ = 1.0 \text{ m}[1 + 18] \]

\[ = 19 \text{ m} \]

160. ANS:
(a) \[ W = - \Delta E_g \]
\[ \Delta E_g = E_{g_2} - E_{g_1} \]
\[ = - \frac{GMm}{r_e} - \left( \frac{GMm}{r_1} \right) \]
\[ = \frac{GMm}{r_e r_1} - (r_e - r_1) \]
\[ = \frac{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \left( 5.98 \times 10^{24} \text{ kg} \right) \left( 1.2 \times 10^3 \text{ kg} \right) (-2.0 \times 10^6 \text{ m})}{(6.37 \times 10^6 \text{ m})(8.37 \times 10^6 \text{ m})} \]
\[ = -1.8 \times 10^{10} \text{ J} \]

But, \[ W = - \Delta E_g = 1.8 \times 10^{10} \text{ J} \]

(b) \[ v = \sqrt{\frac{2E_k}{m}} \]
\[ = \sqrt{\frac{(2)(1.8 \times 10^{10} \text{ J})}{1.2 \times 10^3 \text{ kg}}} \]
\[ = 5.5 \times 10^3 \text{ m/s} \]
(a) \( r_0 = r_e + 400 \text{ km} \)
\[
= 6.37 \times 10^6 \text{ m} + 0.40 \times 10^6 \text{ m} \\
= 6.77 \times 10^6 \text{ m}
\]

\[
E_g = \frac{-GMm}{r_0} \\
= \frac{-6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2 \times 5.98 \times 10^{24} \text{ kg} \times 2.00 \times 10^6 \text{ kg}}{6.77 \times 10^6 \text{ m}} \\
= -1.18 \times 10^{11} \text{ J}, \text{ or } -1.18 \times 10^{11} \text{ J}
\]

(b) For orbit:
\[
E_k = \frac{1}{2} \left| E_g \right| \\
= 5.89 \times 10^{10} \text{ J}
\]

(c) \( E_T = E_g + E_k \)
\[
= -1.18 \times 10^{11} \text{ J} + 5.89 \times 10^{10} \text{ J} \\
= -5.89 \times 10^{10} \text{ J}
\]

(d) At perigee:
\( r_p = 6.37 \times 10^6 \text{ m} + 0.28 \times 10^6 \text{ m} \)
\[
= 6.65 \times 10^6 \text{ m}
\]

\[
E_g = \frac{-GMm}{r_p} \\
= \frac{-6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2 \times 5.98 \times 10^{24} \text{ kg} \times 2.00 \times 10^3 \text{ kg}}{6.65 \times 10^6 \text{ m}} \\
= -1.19 \times 10^{11} \text{ J}
\]
\[ E_k = E_T - E_g \]
\[ = -5.89 \times 10^{10} \text{ J} - \left(-1.199 \times 10^{11} \text{ J}\right) \]
\[ = 6.10 \times 10^{10} \text{ J} \]

\[ v = \sqrt{\frac{2E_k}{m}} \]
\[ = \sqrt{\frac{2 \times 6.10 \times 10^{10} \text{ J}}{2.00 \times 10^3 \text{ kg}}} \]
\[ = 7.81 \times 10^3 \text{ m/s} \]

**ANS:**
(a) \( r_0 = r_e + 200 \text{ km} \)
\[ = 6.37 \times 10^6 \text{ m} + 2.00 \times 10^6 \text{ m} \]
\[ = 6.57 \times 10^6 \text{ m} \]

\[ E_g = -\frac{GM_v m}{r_0} \]
\[ = \frac{-6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \times 5.98 \times 10^{24} \text{ kg} \times 5.0 \times 10^2 \text{ kg}}{6.57 \times 10^6 \text{ m}} \]
\[ = -3.03 \times 10^{10} \text{ J} \]

(b) In orbit:
\[ E_k = \frac{1}{2} |E_g| \]
\[ = 1.52 \times 10^{10} \text{ J} \]

(c) \( E_T = E_k + E_g \)
\[ = 1.52 \times 10^{10} \text{ J} - 3.03 \times 10^{10} \text{ J} \]
\[ = -1.52 \times 10^{10} \text{ J} \]
Therefore, the binding energy is $1.52 \times 10^{10}$ J.

(d) At Earth’s surface:
\[
E_{\text{g}} = \frac{-GM_{\text{e}} m}{r_{\text{e}}}
\]
\[
= \left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right)\left(5.98 \times 10^{24} \text{ kg}\right)\left(5.0 \times 10^{2} \text{ kg}\right)
\]
\[
= \frac{6.37 \times 10^6 \text{ m}}{}
\]
\[
= -3.13 \times 10^{10} \text{ J}
\]

For orbit:
\[
E_{x} = E_{T} - E_{g}
\]
\[
= -1.52 \times 10^{10} \text{ J} - \left(-3.13 \times 10^{10} \text{ J}\right)
\]
\[
= 1.61 \times 10^{10} \text{ J}
\]

For escape:
\[
E_{x} = E_{T} - E_{g}
\]
\[
= 0 - \left(-3.13 \times 10^{10} \text{ J}\right)
\]
\[
= 3.13 \times 10^{10} \text{ J}
\]

% increase in launching energy = \[
\frac{(3.13 - 1.61) \times 10^{10} \text{ J}}{1.16 \times 10^6 \text{ J}} \times 100%
\]
\[
= 94%
\]

REF:  K/U
OBJ:  6.3
LOC:  EM1.07
KEY:  FOP 10.8, p.404
MSC:  P

163. ANS:
(a) For escape from the Sun’s surface
\[
\frac{1}{2} m v^2 > \frac{GM_e m}{r_s} \\

v > \sqrt{\frac{2GM_e}{r_s}} \\

> \sqrt{\left(\frac{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2}{6.96 \times 10^8 \text{ m}}\right) \left(1.98 \times 10^{30} \text{ kg}\right)} \\

> \sqrt{37.95 \times 10^{10} \text{ (m/s)}^2} \\

> 6.16 \times 10^5 \text{ m/s}
\]

(b) For escape from Earth’s surface:
\[
\frac{1}{2} m v^2 > \frac{GM_e m}{r_1} + \frac{GM_e m}{r_e}
\]

where \(r_1\) is the distance from the Sun to Earth \((1.49 \times 10^{11} \text{ m})\)

\[
v > \sqrt{2G \left(\frac{M_e}{r_1} + \frac{M_e}{r_e}\right)} \\

> \sqrt{\left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2\right) \left(1.98 \times 10^{30} \text{ kg} + 5.98 \times 10^{24} \text{ kg}\right)} \\

> \sqrt{\left(1.33 \times 10^{-10} \text{ N} \cdot \text{m/kg}^2\right) \left(13.29 \times 10^{18} \text{ kg/m} + 0.94 \times 10^{18} \text{ kg/m}\right)} \\

> 4.34 \times 10^4 \text{ m/s}
\]

REF: K/U OBJ: 6.3 LOC: EM1.07 KEY: FOP 10.8, p.404
MSC: P
ANS:

(a) At the Moon’s surface:
\[
\frac{E_{e_1}}{r_1} = \frac{-GM_m m}{r_m}
\]

At an altitude of \(r_m\):
\[ E_{g_2} = \frac{-GM_m m}{2r_m} \]

Therefore, the initial kinetic energy needed to move from \( r_m \) to \( 2r_m \) is:

\[
E_k = E_{g_2} - E_{g_1} = \frac{-GM_m m}{2r_m} + \frac{GM_m m}{r_m}
\]

\[
\frac{1}{2} mv^2 = \frac{GM_m m}{r_m} - \frac{GM_m m}{2r_m}
\]

\[
v^2 = 2GM_m \left( \frac{1}{r_m} - \frac{1}{2r_m} \right)
\]

\[
v = \sqrt{\left[ 2 \left( 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2 \right) \left( 6.7 \times 10^{22} \text{ kg} \right) \right] \left( \frac{2 - 1}{(2) \left( 1.6 \times 10^6 \text{ m} \right)} \right) ^2}
\]

\[v = \sqrt{2.793 \times 10^6 \text{ (m/s)}^2}
\]

\[= 1.67 \times 10^3 \text{ m/s}, \text{ or } 1.7 \times 10^3 \text{ m/s} \]

(b) On the Moon:

\[m g_m = \frac{GM_m m}{r_m^2} \]

\[g_m = \frac{GM_m}{r_m^2} \]

\[
= \left( 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2 \right) \left( 6.7 \times 10^{22} \text{ kg} \right) \left( 1.6 \times 10^6 \text{ m} \right)^2
\]

\[= 1.74 \text{ N/kg} \]
Then, \( m g_s \Delta h_s = m g_a \Delta h_a \)

\[
\Delta h_a = \frac{g_s \Delta h_s}{g_a}
\]

\[
= \frac{(9.8 \text{ N/kg})(2.0 \text{ m})}{(1.74 \text{ N/kg})}
\]

\[
= 11 \text{ m}
\]

Momentum is conserved in the collision,

\[ m_1 v_1 = (m_1 + m_2) v_{12} \]

where \( v_{12} \) is the initial speed of the pair, starting up the slope.

\[
(1500 \text{ kg})(20 \text{ m/s}) = (1500 \text{ kg} + 500 \text{ kg}) v_{12}
\]

\[
v_{12} = 15 \text{ m/s}
\]

Using conservation of energy,

\[
\frac{1}{2} (m_1 + m_2) v_{12}^2 = (m_1 + m_2) g \Delta h
\]

\[
\Delta h = \frac{v_{12}^2}{2g}
\]

\[
= \frac{(15 \text{ m/s})^2}{2(9.8 \text{ N/kg})}
\]

\[
= 11.5 \text{ m}
\]

\[
\Delta d = \frac{\Delta h}{\sin 30^\circ}
\]

\[
= \frac{11.5 \text{ m}}{0.5}
\]

\[
= 23 \text{ m}
\]
For the spring:

\[ k = \frac{F}{x} \]

\[ = \frac{3mg}{3} \]

\[ = mg \]

Total energy is conserved:

\[ E_{\text{total},1} = E_{\text{total},2} \]

\[ E_k + E_e + E_g = E_k + E_e + E_g \]

\[ 0 + 0 + mgh_1 = 0 + \frac{1}{2}kx_2^2 + 0 \]

\[ mgh_1 = \frac{1}{2}kx_2^2 \]

\[ = \frac{1}{2}mg(3.0 \text{ m})^2 \]

\[ h_1 = 4.5 \text{ m} \]

(a) \[ W = \text{area under curve from } x = 0 \text{ to } x = 0.6 \text{ m} \]

\[ = \frac{1}{2}(0.4 \text{ m})(4 \text{ N}) \cdot \frac{(4 \text{ N} + 8 \text{ N})}{2} \]

\[ = 0.8 \text{ J} + 1.2 \text{ J} \]

\[ = 2.0 \text{ J} \]
(b) \( \Delta E = W = 2.0 \text{ J} \)

(c) \( \Delta E_s = E_s_2 - E_s_1 \)
\[ = 0.8 \text{ J} - 2.0 \text{ J} \]
\[ = -1.2 \text{ J} \]

\( \Delta E_k = -\Delta E_s = 1.2 \text{ J} \)

\[ v = \sqrt{\frac{2 \Delta E_k}{m}} \]
\[ = \sqrt{\frac{(2)(1.2 \text{ J})}{5.0 \text{ kg}}} \]
\[ = 0.69 \text{ m/s} \]

REF: K/U  OBJ: 4.5  LOC: EM1.05  KEY: FOP 10.8, p.400

MSC: P

168. ANS:

(a) \( E_g = -\frac{GM_Em}{r} \)
\[ = -\left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right)\left(5.98 \times 10^{24} \text{ kg}\right)\left(1.00 \times 10^4 \text{ kg}\right) \]
\[ = \frac{1.00 \times 10^8 \text{ m}}{1.00 \times 10^{10}} \]
\[ = -3.99 \times 10^8 \text{ J} \]

(b) For escape, \( E_T > 0 \)
\[ \therefore E_T > 3.99 \times 10^8 \text{ J} \]

(c) To escape:
\[ \frac{1}{2}mv^2 > 3.99 \times 10^8 \text{ J} \]
\[ v > \sqrt{\frac{(2)(3.99 \times 10^8 \text{ J})}{1.00 \times 10^4 \text{ kg}}} \]
\[ > 2.82 \times 10^3 \text{ m/s} \]

REF: K/U  OBJ: 6.3  LOC: EM1.07  KEY: FOP 10.8, p.404

MSC: P