

2006-07 Grade 11 CHAMP Math Contest

Part A: (5 credits each)

1. If $c^d = 3$, then $c^{4d} - 5$ equals:

- a) 7 b) 22 c) 76 d) 86 e) none of these

2. The expression $8^9 + 8^9 + 8^9 + 8^9 + 8^9 + 8^9 + 8^9 + 8^9$ in simplest power form is:

- a) 8^{72} b) $8(9^8)$ c) 8^{10} d) 64^9 e) 72^8

3. What is the least positive integer by which 28 should be multiplied so that the product is a perfect cube?

- a) 14 b) 42 c) 84 d) 98 e) 196

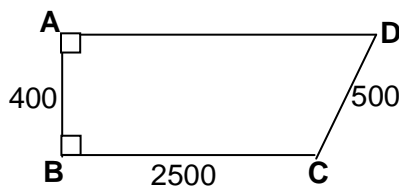
4. Two ships leave New York at the same time. The course of the first ship makes an angle of 60° with the course of the second ship. If each ship is going 15 km/h, then the distance, in km, between them at the end of two hours is:

- a) 15 b) 30 c) 45 d) 60 e) none of these

5. The distance from $P(a, b)$ to $Q(-a, 3b)$, in simplest form is:

- a) $\sqrt{2a^2 + 2b^2}$ b) $4b$ c) $4\sqrt{a^2 + b^2}$ d) $2\sqrt{a^2 + b^2}$ e) $2(a + b)$

6. A farm has dimensions, in metres, as shown on the diagram.
If $AD > BC$, the perimeter of the farm, in metres, is:



- a) 5600 b) 6400 c) 5900 d) 6200 e) cannot be determined

7. The lines $x = 0$, $y = 0$, and $2x + y = 4$ form a triangle. The number of points with integer co-ordinates which are inside this triangle is:

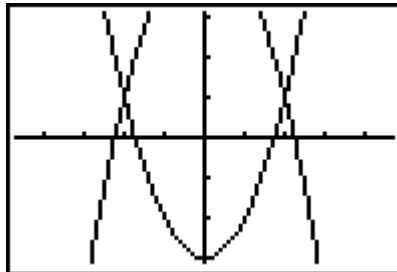
- a) 1 b) 2 c) 3 d) 4 e) more than 4

2006-07 Grade 11 CHAMP Math Contest

8. The parabola $y = 5x^2 + 5x - 60$ intersects the x-axis at points P and Q. The length of segment PQ is:

- a) 1 b) 7 c) 11 d) 13 e) 35

9. The quadratic relations $y = x^2 - 3$ and $y = -x^2 + k$ intersect at the points $(-2,1)$ and $(2,1)$. The value of "k" is:



- a) -3 b) 1 c) 3 d) 4 e) 5

10. Assume the Earth is a sphere of circumference 40 000 km. If a belt is placed around the equator one metre away from the Earth at all points, how much greater than the circumference of the earth would the length of the belt be?

- a) π m b) 2π m c) 40 000 m d) 40 000 km e) none of these

Part B: (6 credits each)

11. If the reciprocal of $x+1$ is $x-1$, then x is equal to:

- a) 0 b) 1 c) -1 d) ± 1 e) none of these

12. The quantity which, when divided by $2x+4$ gives a quotient of $x-2$ and a remainder of 9, is:

- a) $2x^2 - 8$ b) $2x^2 + x + 8$ c) $2x^2 - x + 1$ d) $2x^2 + 4$ e) $2x^2 + 1$

13. If $a, b,$ and c are such that $a - b = b - c = 3$, then $a^2 - 2b^2 + c^2$ has the value:

- a) 12 b) $12b$ c) 15 d) 18 e) none of these

2006-07 Grade 11 CHAMP Math Contest

14. The points $A(1,1)$, $B(0,0)$, and $C(2,-1)$ define a triangle in the Cartesian plane. The value of angle B is:

- a) 1.25° b) 18.43° c) 50.77° d) 71.57° e) 108.43°

15. The last digit in the expansion of $(7^{11})^6$ is:

- a) 5 b) 9 c) 7 d) 1 e) 3

16. The points $(1, y_1)$ and $(-1, y_2)$ lie on the parabola $y = px^2 + qx + 5$. If $y_1 + y_2 = 14$, then the value of p is:

- a) 2 b) 5 c) 7 d) $2 - q$ e) none of these

17. The lengths of the sides of a triangle are 12, 35 and 37 units. The shortest distance from the longest side to the opposite vertex is:

- a) $\frac{144}{37}$ b) $\frac{420}{37}$ c) $\frac{1225}{37}$ d) $\frac{444}{35}$ e) $\frac{1295}{12}$

18. A student makes a trip in three hours. Two-thirds of the distance travelled was by car and the remainder by boat. If the car's speed was three times the speed of the boat, how many hours were spent travelling by boat?

- a) 1.2 b) 1 c) 1.8 d) 2 e) 0.75

19. The lengths of the hour and minute hands of a clock are 4 cm and 6 cm respectively. The distance, in cm, between the tips of the hands at two o'clock is:

- a) $\sqrt{52 - 24\sqrt{3}}$ b) $2\sqrt{7}$ c) $2\sqrt{5}$ d) $2\sqrt{3}$ e) $2\sqrt{2} - \sqrt{3}$

20. For some integer, x , the value of $1 + \frac{1}{x + \frac{1}{x}}$ is $\frac{31}{26}$. The value of x is:

- a) 285 b) 57 c) 31 d) 26 e) 5

2006-07 Grade 11 CHAMP Math Contest

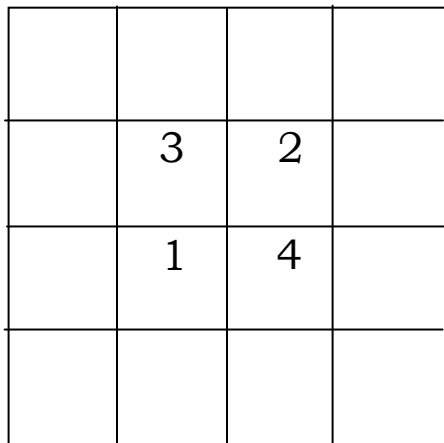
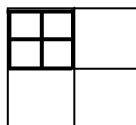
Part C: (8 credits each)

21. A circular piece of metal of maximum size is cut from a square piece, and then a square piece of maximum size is cut from the circular piece. The total amount of metal wasted is:

- a) $\frac{1}{4}$ the area of the original square b) $\frac{1}{2}$ the area of the original square c) $\frac{1}{2}$ the area of the circular piece d) $\frac{1}{4}$ the area of the circular piece e) none of these

22. To solve the CHAMPDOKU puzzle shown below, one of the digits 1, 2, 3 and 4 must be placed in each cell in such a way that each of the four digits must appear:

- (i) once in each row
 (ii) once in each column
 (iii) once in each 2 X 2 corner grid



The number of possible solutions to this puzzle is:

- a) 3 b) 4 c) 5 d) 6 e) 7

23. For what values of r does the line $y = 2x - 3$ intersect (at one point or more) the circle $x^2 + y^2 = r^2$?

- a) $r \geq 1.34$ b) $r \geq 1.5$ c) $r \geq 1.8$ d) $r \geq 3$ e) $r \geq 6$

2006-07 Grade 11 CHAMP Math Contest

24. If the perimeter of a rectangle is " p " and its diagonal is " d ", then the difference between the length and width of the rectangle is:

- a) $\frac{\sqrt{8d^2 - p^2}}{2}$ b) $\frac{\sqrt{8d^2 + p^2}}{2}$ c) $\frac{\sqrt{6d^2 - p^2}}{2}$ d) $\frac{\sqrt{6d^2 + p^2}}{2}$ e) $\frac{\sqrt{8d^2 - p^2}}{4}$

25. An airplane leaves an aircraft carrier and flies due south at 400 km/h. The carrier proceeds 60° west of north at 32 km/h. If the plane has enough fuel for 5 hours of flying, what is the maximum distance south the pilot can travel so that the fuel remaining will allow a safe return to the carrier?

- a) 2.39 km b) 709.71 km c) 955.38 km d) 967.69 km e) 1044.80 km