Lesson 8: Using Transformations and Mapping to Graph Functions of the Form $y = af[k(x-d)] + c$

**Part A - An Overview**

**Vertical Stretch/Compression**
- If $|a| > 1$, stretch
- If $0 < |a| < 1$, compression
- If $a > 0$ (positive), same direction as parent function
- If $a < 0$ (negative), reflected in the $x$-axis

**Vertical Translation (shift) Up/Down**
- If $c > 0$, shift up
- If $c < 0$, shift down

**Horizontal Stretch/Compression**
- If $|k| > 1$, compression
- If $0 < |k| < 1$, stretch
- If $k > 0$ (positive), same direction as parent function
- If $k < 0$ (negative), reflected in the $y$-axis

**Horizontal Translation (shift) Left/Right**
- If adding, shift left
- If subtracting, shift right

$y = af[k(x-d)] + c$

**Remember:** “$k$” and “$d$” affect $x$, “$a$” and “$c$” affect $y$ !!!!

**Example:**

$$y = 4\sqrt{-2(x-3)} - 1$$

- Vertical stretch by a factor of 4
- The square root function
- Horizontal translation 3 units right
- Vertical translation 1 unit down
- Reflection in the $x$-axis
- Horizontal compression by a factor of $1/2$
- Reflection in the $y$-axis
Part B – Graphing using Transformations

Transformations **MUST** be applied in the following order:
1st - Horizontal \((k)\) and vertical \((a)\) stretches/compressions/reflections
2nd - Horizontal \((d)\) and vertical translations \((c)\) (left/right and up/down)

Example 1: If \(f(x) = |x|\), state the transformations in words, graph the function \(y = 2f(x - 3) - 2\) and state the domain and range.

![Graph of function](image)

**Textbook shows a table for transformations.**

<table>
<thead>
<tr>
<th>(x, y)</th>
<th>Domain:</th>
<th>Range:</th>
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<tbody>
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Part C - Mapping

What if you were asked to graph: \(y = -3 \sqrt{\frac{1}{2}(x + 4) + 2}\)

It can get quite complicated when doing each transformation on the graph.

Another method is called **Mapping**

Given \(y = af(k(x - d)) + c\), each point from the parent function can be translated using mapping:

\[
(x, y) \rightarrow \left( \frac{x}{k} + d, ay + c \right).
\]
Example 2: The point (2, 6) is on the graph of \( y = f(x) \). Determine the corresponding coordinates of the point on each of the following graphs.

a) \( y = 2f(x + 3) \)  
b) \( y = -f(2x) - 5 \)

**Part D – Steps for Graphing using Mapping**

1. Graph the parent function.
2. Rewrite the equation in the form \( y = af(k(x - d)) + c \)
3. Locate at least three points on the parent function and use the mapping \((x, y) \rightarrow \left(\frac{x}{k} + d, ay + c\right)\) to determine three points on the transformed function.
4. Use mapping to determine the location of any asymptotes

**Part E – Applying a Combination of Transformations**

Example 3:
Graph \( y = -3\sqrt{\frac{1}{2}(x + 4)} + 2 \), state the transformations in words and state the domain and range. (Sketch the parent function also.)

\[
\begin{array}{|c|c|}
\hline
(x, y) & \\
\hline
\hline
\hline
\hline
\end{array}
\]

Domain: Range:
Example 4: If \( f(x) = \frac{1}{x} \), sketch the graph of \( y = f\left[-\frac{1}{2}(x + 2)\right] - 3 \). 

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Domain: __________________ Range: __________________

Example 5:

i) Explain what transformations you would need to apply to the graph of \( y = f(x) \) to graph each function.

ii) State the function with the transformations for each parent function.

a) \( y = 5f(x) - 3 \)

b) \( y = \frac{1}{4} f\left[-\frac{1}{2}(x - 3)\right] \)

ii) Quadratic: ii) Quadratic: 

Reciprocal: Reciprocal: 

Square Root: Square Root: 

Absolute Value: Absolute Value: